MTE101

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**B.Sc. (Semester - V) (CBCS Ordinance)**
**EXAMINATION APRIL 2023**
**Mathematics**
**Foundation of Mathematics**

**[Duration: 02 Hours]** **[Total Marks:80]**

**Instructions:** 1) All questions are **Compulsory** However **Internal** choice are available.
2) Figures to the **Right** indicate full marks.
3) Use of scientific non-programmable calculators is allowed.

**Q.1** Answer **Any Four** of the following: - **16**

**a)** When is a sequence in $\mathbb{R}$ said to be convergent? Write your answer using quantifiers.

**b)** For $a, b \in \mathbb{Z}$, Prove that $a^2 - 4b \neq 2$.

**c)** Identify the set $A = \{x \in \mathbb{R}; ||x - 3| - |x - 4|| = 2\}$

**d)** Examine if the function $f: \mathbb{R} \to \mathbb{R}$ *given by* $f(x) = x + [x]$ is one-one and /or onto; [x] denotes the greatest integer not greater than x.

**e)** $f: X \to Y$ is a function and $\{A_\lambda ; \lambda \in \wedge\}$ is a family of subsets of Y

$$\textit{show that } f^{-1}\left(\bigcap_{\lambda\in\wedge} A_\lambda\right) = \bigcap_{\lambda\in\wedge} f^{-1}(A_\lambda)$$

**f)** *Let* $\sim$ be an equivalence relation on X show that any two equivalence classes are identical or disjoint.

**Q.2** Answer **Any Four** of the following: - **16**

a) Let $x \in \mathbb{R}$ *and* $x > -1$. Use induction principle to prove that $(1 + x)^n \geq 1 + nx$ for $n \in \mathbb{N}$.

b) Show that every integer $n \geq 2$ is a product of primes.

c) For $n \in \mathbb{N}$ let In denote the set {1,2……n} if A is any set and $f: A \to I_n$ is a one-one map, prove that A is finite and $|A| \leq n$.

d) Show that any infinite subset of $\mathbb{N}$ is countably infinite.

e) Examine if the following relations on $\mathbb{N}$ are partial orders: -

i) $R_1$ given by $m\, R_1 n$ if and only if m divides n.

ii) $R_2$ given $mR_2n$ if and only if $m \leq 2n$.

f) Define the terms ' Maximal element' and minimal element' of subset A of partially ordered set $(x, \leq)$.Give an example to show that a set A any have more than one minimal element and more than one maximal element.

**Q.3** **A)** Answer **Any One** of the following: - **06**

**a)** Prove or disprove the following: -

i) There is no rational number such that $x^2 = 10$ .

ii) For a Non-zero real number $x, \left(x + \frac{1}{x}\right)^2 \geq 4$.

**b)** Identify the following sets: -

i) $\bigcup_{n \in \mathbb{Z}} J_n$ ; where $J_n = (n, n+1)$

ii) $\bigcap_{n \in \mathbb{N}} F_n$ ; where $F_n = (o, 1/n)$

iii) $\bigcup_{x \in \Lambda} I_x$ ; where $\Lambda = (0,1) and\ I_x = (-\infty, x)$

**B)** Identify the set $S = \{(x, y) \in \mathbb{R}^2 ; |x| \leq |y|\}$ Represent the set by a shaded Region in the co-ordinate plane $\mathbb{R}^2$ . **06**

**Q.4** **A)** Answer **Any One** of the following: - **06**

**a)** $f: X \to Y$ be a function and A,B be any subsets of X show that $f(A \cap B) \leq f(A) \cap f(B)$. Further show that F is injective if and only if $f(A \cap B) = f(A) \cap f(B)$ for all subsets A, B of X.

**b)** $f: \mathbb{R}^2 \to \mathbb{R}^2$ is given $f(x, y) = (ax + by,\ cx + dy)$, where $a, b, c,$ $d \in \mathbb{R}\ and\ ad - bc \neq 0$ show that f is a bijection and find $f^{-1}$.

**B)** A relation $\sim$ on $\mathbb{R}$ is given by $x \sim y$ if and only if $[x] = [y]$; where [x] denotes the greatest integer not greater than x. show that $\sim$ is an equivalence relation on $\mathbb{R}$. Write the quotient set and find a transversal for the relation $\sim$. **06**

**Q.5** **A)** Answer **Any One** of the following: - **06**

**a)** State (i) The Induction principle and (ii) The well-ordering Principle show that (i) and (ii) are equivalent.

**b)** State 'The well –ordering Principle and use it to prove that

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}.$$

**B)** Prove the following: - **06**

**i)** A countable union of countable sets is countable

**ii)** Cartesian product of any two countable sets is countable

**Q.6** A) Answer **Any One** of the following: - **06**

**a)** Show that $\mathbb{R}$ is uncountable.

**b)** Show that $\mathbb{N} \times \mathbb{N}$ is countable. Hence deduces that $Q^+$, the set of all positive rationales is countable.

**B)** **(i)** Define the term :- A chain in a partially ordered set $(x, \leq)$,Show that a maximal element in a chain is a maximum. **06**

**(ii)** Let $X = \{2,3,4,6,9,12,18,36\}$ and the order relation $\leq$ *on* $X$ be defined as $a \leq b$ if and only if a is a multiple of b. Draw Mass Diagram for $(x, \leq)$. Find LUB of $A = \{3,4,6\}$ if it exists.