MTC107

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B.Sc. (Semester-V) Course (CBCS Ordinance)
EXAMINATION APRIL 2023
MATHEMATICS
Calculus of 2 & 3 Variable

[Duration: 3 Hours] **[Max. Marks: 120]**

Instructions: 1) **All** Questions Are **Compulsory.** However **Internal** choice is available.
2) Figures to the **Right** indicate full Marks to each question / Sub question.
3) Use of Scientific/non-programmable calculators is allowed.

Q.1. Answer **Any Five** of the following: **[5×4=20]**

a) Use $\varepsilon - \delta$ def^n to show that $\lim\limits_{(x,y)\to(0,0)} \frac{xy(x^2-y^2)}{x^2+y^2} = 0.$

b) Find the directional derivative of the function $\phi = x^2 - y^2 + 2z^2$ at the point P(1,2,3) in the direction of the line PQ where Q is the point (5,0,4).

c) Investigate the continuity at (0,0) of

$$f(x,y) = \frac{x^2-y^2}{x^2+y^2} \quad , \quad (x,y) \neq (0,0)$$
$$= 0 \quad , \quad (x,y) = (0,0).$$

d) Find the partial derivatives of f at (0,0) of

$$f(x,y) = \frac{xy}{x^2+y^2} \quad , \quad (x,y) \neq (0,0)$$
$$= 0 \quad , \quad (x,y) = (0,0).$$

e) If $z = e^{xy^2}$, $x = t\cos t$, $y = t\sin t$

Compute $\frac{dz}{dt}$ at $t = \pi/2$.

f) For $\bar{f} = y(x+z)\,i + z\,(x+y)j + x(y+z)K$

Find curl ($Curl\,\bar{f}$).

g) Compute the equation of the plane tangent to the surface $3xy + z^2 = 4$ at the point (1,1,1).

Q.2. Answer **Any Five** of the following: **[5×4=20]**

a) State the fundamental theorem of Calculus.

b) Find the spherical coordinates of the point (1,-1,1) and the Cartesian coordinates of $(3, \pi/6 , \pi/4)$

c) Find the volume of the ball $x^2 + y^2 + z^2 = R^2$ using Spherical coordinates.

d) Find the area of the domain enclosed by $y = \sqrt{x}$, $y = 2\sqrt{x}$, $x = 4$.

e) In case of multiple integration, explain the working of the cavaliers Principle.

f) Find the volume under the graph of $f(x,y) = x^2 + y^2$ between the planes $x = 0$, $x = 3$, $y = -1$ and $y = 1$.

g) Define the Moment of Inertia about the x-axis.

Q.3. **A.** Answer **Any One** of the following: **[5 marks]**

a) i) Find $\lim\limits_{(x,y)\to(0,0)} \frac{x^3+2x^2+xy^2+2y^2}{x^2+y^2}$.

ii) For the implicit function $e^{x-y} + x^2 - y = 1$ find $\frac{dy}{dx}$ at $x = 0$, $y = 0$. **[5 marks]**

OR

b) i) Show that $\lim\limits_{(x,y)\to(0,0)} \frac{\partial}{\partial x}\sqrt{x^2+y^2}$ does not exists. **[5 marks]**

ii) $f(x,y) = Log\sqrt{x^2+y^2}$, $find\ \nabla f\ (x,y)$. **[5 marks]**

B. Show that the function $f(x,y) = \frac{x^3-y^3}{x^2+y^2}$, $(x,y) \neq (0,0)$ **[10 marks]**

$= 0$, $(x,y) = (0,0)$

Is continuous at $(0,0)$ possesses $f_x(0,0)$ & $f_y(0,0)$ but is not differentiable at $(0,0)$.

Q.4. **A.** Answer **Any One** of the following:

a) i) Investigate the maxima and minima of the function $f(x,y) = x^2 - 3xy^2 + 2y^4$. **[6 marks]**

ii) Expand $x^2y + 3y - 2$ in powers of $(x-1)$ and $(y+2)$. **[4 marks]**

OR

b) i)If $\bar{u} = \bar{w} \times \bar{r}$, where $\bar{w}$ is a constant vector and $\bar{r} = xi + yj + zk$ show that $\frac{1}{2}$ curl $\bar{v} = \overline{w}.$ **[5 marks]**

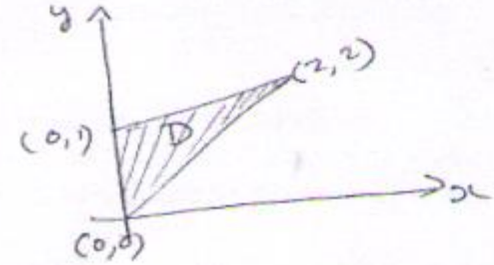
ii) Show that $\nabla^2 f(r) = f''(r) + \frac{2}{r}y'(r)$. **[5 marks]**

B. Find the stationary points of the function xy^2z^2 subject to the condition. **[10 marks]**

$x + y + z = b$, $x > 0$, $y > 0$, $z > 0$.

Q.5. **A.** Answer **Any One** of the following:

a) i) Calculate the integral of $f(x, y) = (x + y)^2$ over the region as shown in fig: **[5 marks]**



ii) Evaluate $\iiint xyz\, dx\, dy\, dz$ over the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$, using the spherical polar coordinates. **[5 marks]**

OR

b) i) Evaluate $\iiint \frac{1}{(x+y+z+1)}\, dx\, dy\, dz$ over the region bounded by the planes $x = 0\,, y = 0\,, z = 0\,, x + y + z = 1$ **[5 marks]**

ii) Evaluate $\iint x^2y^2dxdy$ over the circular area $x^2 + y^2 \leq 1$ **[5 marks]**

B. i) Find the centre of mass of the rectangle [0,1] x [0,1] if the mass density is e^{x+y}. **[5 marks]**

ii) Using the transformation $x + y = u\,, y = uv$

Show that $\int_0^1 dx \int_0^{1-x} \frac{y}{e^{x+y}}\, dy = \frac{1}{2}(e - 1)$ **[5 marks]**

Q.6. **A.** Answer **Any One** of the following:

a) i) For $A = (2x - y)i - yz^2j - y^2zK$, δ is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundry. Use Stoke's theorem to evaluate $\oint_c A.dr$ **[5 marks]**

ii) Let $f(x, y) = yi - xj$ and Let c be a circle of radius r traversed counter clockwise. **[5 marks]**

Write the line integral $\int_c f(s).ds$ as a double integral using Green's theorem and Evaluate.

OR

b) i) Show that $f = (2xy + z^3)i + x^2y + 3x\,z^2k$ is conservative and evaluate $\int f.dr$ along any curve jointing (1,-2,1) and (3,1,4) **[5 marks]**

ii) If $f = 2xyi - yzj + x^2k$

Evaluate $\int_S f.n\, ds$, where δ denotes the entire surface of the cube bounded by the coordinate planes and the planes $x = a, y = a\,, z = a$ by the application of Gauss's theorem. **[5 marks]**

[10 marks]

B. Prove that if $\int_{P_1}^{P_2} f.dr$ is independent of the path joining any two points $P_1 and\ P_2$ is a given region , then $\oint f.dr = 0$ for all closed paths in the region and conversely.