UMTC108

Total No. of Printed Pages: 04

B. Sc. Course (CBCS) Ordinance Sem-VI
EXAMINATION APRIL 2023
Mathematics - Differential Equations II

[Time: 3 Hours] **[Max. Marks:120]**

Instructions: 1. All questions are compulsory.
2. Figures to the **right** indicate full marks.
3. Use of non-programmable calculators is allowed.
4. Symbols have their usual meanings.

Q1 Attempt **any five** of the following: **20**

a) $Solve\ \frac{y}{x}\frac{dy}{dx} + \frac{x^2+y^2-1}{2x^2+2y^2+1} = 0$

b) Obtain the Legendre's polynomials
$P_0(x), P_1(x), P_2(x)$.
Use any approach.

c) Let ϕ be any solution for $x > 0$ of the Bessel equation of order α and put $\psi(x) = \sqrt{x}\phi(x)$. Show that ψ satisfies the equation.

$$y'' + \left[1 + \frac{\frac{1}{4} - \alpha^2}{x^2}\right] y = 0$$

d) Prove that $J_{-\frac{1}{2}(x)=\sqrt{\frac{2}{\pi x}}\cos(x)}$

e) Consider the initial value problem

$$\frac{dy}{dx} = 1 + xy, y(0) = 1$$

Compute the first four approximations to the solution using Picards method.

f) State the following differential equations:
i) Euler equation
ii) Legendre equation

g) Consider the equation

$$x^2y'' + 2xy' - n(n+1)y = 0,$$

where n is a non-negative integer. Show that infinity is a regular singular point.

Q2 Attempt **any five** of the following: **20**

a) $\frac{dy}{dx} - x\tan(y-x) = 1$

b) *Prove that* $P_{2n}(0) = \frac{(-1)^n(2n)!}{2^{2n}(n!)^2}$ *and* $P_{2n+1}(0) = 0$

c) *Prove that* $J_{n+1}(x) + J'_n(x) = \frac{n}{x}J_n(x)$

d) *Express* $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials.

e) Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with the boundary conditions $y = 1$ when $x = 0$, find approximately y for $x = 0.1$ by using Euler's modified method. Take $h = 0.02$.

f) Solve in series the equation $y'' + x^2y = 0$

g) Consider the equation $(1-x^2)y'' - xy' + p^2y = 0$, where p is a non-negative constant. Compute the indicial polynomial and find its two roots.

Q3 A) Attempt **any one** of the following:

a) i) Solve $x\frac{dy}{dx} + y = x^3y^6$ **05**

ii) Find particular solution of $y'' - 6y' + 9y = \frac{e^{3x}}{x}$ by the method of variation of parameters. **05**

b) i) Solve $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$ **05**

ii) Find the general solution of $y'' - 3y' + 2y = (1+x)e^{3x}$ by the method of undetermined coefficients. **05**

B) .

a) If $y_1(x)$ *and* $y_2(x)$ are any two solutions of the equation $y''(x) + P(x)y'(x) + Q(x) = 0$ *on* $I = [a,b]$, then prove that the Wronskian) $W(x)$ is either identically zero or never zero on I. **05**

b) Reduce the following differential equation **05**

$$x^2y'' + pxy' + qy = 0, p, q \in \mathbb{R}$$

into a linear equation with constant coefficients.

Q4 A) Attempt **any one** of the following:

a) i) Prove that $P_n(x) = P'_{n+1}(x) - 2xP'_n(x) + P'_{n-1}(x), n \geq 1$. **05**

ii) Prove that $J_4(x) = \left(\frac{48}{x^3} - \frac{8}{x}\right)J_1(x) + \left(1 - \frac{24}{x^2}\right)J_0(x)$. **05**

b) i) Prove that $\int_{-1}^{1} xP_n(x)\,P'_n(x)dx = \frac{2n}{2n+1}$ **05**

ii) Prove that $x^2\left(\left[J'_n(x)\right]^2 + J_{n+1}(x)J_{n-1}(x)\right) = n^2J^2{}_n(x)$. **05**

B) Obtain two linearly independent generalized power series solutions of the differential equation. **10**

$$2x^2y'' + x(2x+1)y' - y = 0$$

using Frobenius method.

Q5 A) Attempt **any one** of the following:

a) i) Find the inverse Laplace transform of **05**

$$F(s) = \frac{s^2-4}{(s^2+4)^2(s^2+1)}$$

ii) Find the inverse Laplace transform of **05**

$$F(s) = \frac{1}{s(s^2+a^2)} \text{ and } G(s) = \frac{1}{s(s+a)^3}$$

b) i) $Evaluete\ L^{-1}\left(\frac{s^2}{s^4+4}\right)$ **05**

ii) Use transform method to solve **05**

$$x''(t) - 2x'(t) + x(t) = e^t$$

with the initial conditions $x(0) = 2, x'(0) = -1$

B)

a) $Let\ L[f(t)] = F(s)\ and\ H(t-a) = \begin{cases} 0 & if\ t < a \\ 1, & if\ t \geq a \end{cases}, a > 0.\ Prove\ that$ **05**

$$L\{f(t-a)H(t-a)\} = e^{-as}F(s).$$

b) $If\ L[f(t)] = F(s), then\ prove\ that$ **05**

$$L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}(F(s)), \quad n \in \mathbb{N}.$$

Q6 A) Attempt **any one** of the following: **10**

a) Compute *y(0.5)* by Milnes predictor-corrector method from the equation

$$\frac{dy}{dx} = x(x^2+y^2)e^{-x}$$

Given that

x:	0	0.1	0.2	0.3
y:	1	1.005	1.018	1.04

b) Using Adams-Moulton method obtain the solution of

$$\frac{dy}{dx} = x - y^2$$

at x=1 given the values

x:	0	0.2	0.4	0.6
y:	0	0.02	0.0795	0.1762

B) Find y approximately at $x = 0.2$, taking $h = 0.1$ for the initial value problem **10**

$$\frac{dy}{dx} = x + y^2, y(0) = 1$$

Use Runge-Kutta method of 4th order.