

CARMEL COLLEGE OF ARTS, SCIENCE & COMMERCE FOR WOMEN,
NUVEM-GOA
SEMESTER END EXAMINATION, APRIL-MAY 2023 (REPEAT)
Semester: IV of BSC

Course Title: Analytical Geometry Course Code: MTS 102
Total marks: 60 Date: 03/05/2023 Duration: 2hrs Total No of pages: 2

Instructions: 1. All questions are compulsory however internal choice is available.
2. Figures to the right indicate marks allotted to each question/ subquestion.
3. Use of non-programmable calculator is allowed.

Q. 1. Answer any five of the following. (2X5=10)

- Find the radius and the center of the circle $x^2 + y^2 - 4x - 6y + 4 = 0$.
- Show that $4x^2 - 5xy + y^2 + 2x + y - 2 = 0$ represent a pair of straight lines.
- Show that the points $(c, a+b)$, $(a, b+c)$ and $(b, c+a)$ are collinear.
- Verify if the points $(-2,1)$ and $(-1,3)$ lie on the same side of the straight-line $x+2y+5=0$
- Find the equation of the circle passing through the points $(1,0)$, $(-1,0)$ and $(0,-1)$.
- Obtain a straight line passing through $(-3,1)$ which is parallel to the tangent to the circle $x^2+y^2=4$ at $(0,2)$.
- Check if the points $(1,-2)$ and $(1,3)$ lie on the interior or exterior to the circle $x^2 + y^2 - 2x - y + 1 = 0$.

Q. 2. Answer any five of the following. (2X5=10)

- Find the direction ratios and direction cosines of the plane $3x - 2y + z = 5$.
- Show that the points $(0,1,2)$, $(2,-1,3)$ and $(1,-3,1)$ form the vertices of an isosceles right angled triangle.
- Find the length of the chord of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$ intercepted on the straight line $2x+3y=6$.
- Transform the equation in polar form $x^2 + y^2 - 4x = 0$.
- Find a vector of magnitude 5 in the direction of the vector $3\hat{i} - 5\hat{j} + 2\hat{k}$.
- Find the distance of the point $(1,-2,1)$ from the plane $4x - 8y + 4z = 3$.
- Represent the points in polar coordinate system.
i. $(-2, \frac{\pi}{6})$ ii. $(2, 0)$ iii. $(3, -\frac{\pi}{2})$ iv. $(1, \frac{\pi}{4})$

Q.3. Answer the following questions.

- A. Answer any one of the following questions. (5)
- Find the distance of the line $x + 3y = 1$ from the point $(2, -5)$ measured parallel to the line $2x - y + 2 = 0$.
 - Show that the locus of a point P which moves so that $AP:BP = \alpha:\beta$ with respect to two points A $(\beta \cos 2u, \beta \sin 2u)$ and B $(\alpha \cos 2v, \alpha \sin 2v)$ is $x + y \tan(u+v) = 0$.

- B. Show that the line $x + \sqrt{3}y = 8$ touches the circle $x^2 + y^2 = 16$. Find the point of contact. (5)

Q.4. Answer the following questions.

- A. Answer any one of the following questions. (5)

- Show that the equation of the line passing through (α, β) and through the point of intersection of the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is $\frac{a_1x + b_1y + c_1}{a_1\alpha + b_1\beta + c_1} = \frac{a_2x + b_2y + c_2}{a_2\alpha + b_2\beta + c_2}$
- Find the foci, vertices, directrices, transverse and the coordinate axis, eccentricity of the hyperbola given by $x^2 - 9y^2 + 2x + 16y + 11 = 0$.

- B. Find the equation of a line passing through the points $(6, -1, 4)$ and $(2, -3, 1)$. Find the coordinates of the foot of the perpendicular from the point $(1, 1, 1)$ to the line. Also find the length of the perpendicular. (5)

Q.5. Answer the following questions.

- A. Answer any one of the following questions. (5)

- Find the equations of all 3 sides of the triangle formed by the vertices $A(1, -2, 5)$, $B(-1, 3, 1)$ and $C(1, 0, -2)$. Also find the coordinates of the centroid of the triangle.
- What is a conjugate of a hyperbola? If e and e' are eccentricities of a hyperbola and its conjugate, Show that $e^{-2} + e'^{-2} = 1$.

- B. Determine the nature of the conic and hence find eccentricity, directrix, vertices and length of the latus rectum of the ellipse $3x^2 + 3y^2 + 2xy + 16x + 20 = 0$. (5)

Q.6. Answer the following questions.

- A. Answer any one of the following questions. (5)

- Find the distance of the plane $5x - 2y + z = 1$ from the origin and the direction cosines of the plane. Verify if the plane intersects the line joining $(1, 2, 6)$ and $(0, -1, 2)$, if so find the point of intersection.
- Find the equation of the plane passing through the points $A(-2, 1, 1)$, $B(1, -1, 1)$ and $C(1, 1, 2)$. Hence obtain its distance of this plane from the origin.

- B. If P the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2}$ and the plane $x+y+z=6$. Find the coordinates of P. If B $(-1, 5, 1)$, Obtain the length of PB. (5)