CARMEL COLLEGE OF ARTS, SCIENCE & COMMERCE FOR WOMEN, NUVEM-GOA

SEMESTER END EXAMINATION, APRIL-MAY 2023

Sem. IV of B.Sc. Mathematics Class & Div.: **SY B.Sc (A&B)**

Course Title & Code: **Analysis and Operations Research (UMTC104)**

Maximum marks: **80** Date: 02/05/2023 Duration: **2 hrs** Total No of pages: **3**

Instructions: *i) All questions are compulsory.*

ii) Figures to the right indicate the maximum marks.

iii) Non-programmable calculators are allowed.

I. Answer **<u>any 4</u>** of the following. **(4 x 4 = 16)**

a. Show that the series $\sum_{n=0}^{\infty} c$ diverges whenever $c \neq 0$.

b. Discuss the convergence of the following series.

i. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n-1}}$

ii. $\sum_{n=1}^{\infty} \left(\frac{2}{n}\right)^n$

c. Let $u, v: [a, b] \to \mathbb{R}$ be differentiable. Assume that u' and v' are integrable on $[a, b]$. Show that

$$\int_a^b u(x)v'(x)dx = u(x)v(x) \,|_b^a - \int_a^b u'(x)v(x)dx$$

d. For any bounded function $f: I \to \mathbb{R}$, show that the set of all lower sums of f is bounded above.

e. Show that the function $f: [0,1] \to \mathbb{R}$ defined as

$$f = \begin{cases} 0; & x \in \mathbb{Q} \\ 1; & x \in [0,1] \backslash \mathbb{Q} \end{cases}$$

is not integrable on [0,1].

f. Find the lower sum of $f: [0,1] \to \mathbb{R}$ with respect to the partition $P = \left\{0, \frac{1}{n}, \frac{2}{n}, \dots, 1\right\}$ where $f(x) = x \;\forall\; x \in [0,1]$.

II. Answer **<u>any 4</u>** of the following **(4 x 4 = 16)**

a. When is a sequence of functions (f_n), where $f_n: I \to \mathbb{R}$, said to be uniformly Cauchy?

b. Find the pointwise limit of the sequence of functions $f_n: [0,1] \to \mathbb{R}$ where

$$f_n(x) = \begin{cases} nx; & 0 \leq x \leq \frac{1}{n} \\ 1; & x > \frac{1}{n} \end{cases}$$

c. Discuss the nature of convergence of the series $\sum_{n=0}^{\infty} \frac{x^2}{x^2+n^2}$; $x \in [0,1]$.

d. Freshwater is supplied through three reservoirs with daily supply capacities of 15, 20 and 25 million litres of freshwater respectively. On each day supply must be provided to for cities A, B, C and D whose demands are 8, 15, 22 and 15 million litres is given below:

Reservoirs	Cities			
	A	B	C	D
P	2	3	4	5
Q	3	2	5	2
R	4	1	5	2

Find initial basic solution using North-west corner method.

e. Find the dual of: Minimize , $Z = 3x_1 - 2x_2 + 4x_3$

Subject to
$$2x_1 + 5x_2 + 4x_3 = 7;$$
$$x_1 + 3x_3 \leq 4;$$
$$x_2 - x_3 = 2;$$
$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$

f. Customers arrive at a ticket counter at a rate of 50 per hour and tickets are issued in order of their arrival. The average time taken for issuing a ticket is 1 minute. Assuming that customer arrivals follow Poisson distribution and service time follows exponential distribution, find the average number of customers in the system and the average waiting time in the queue.

III. A) Answer **<u>any one</u>** of the following. **(6)**

i. If (a_n), $n \geq 0$, is a sequence of real numbers such that $\sum|a_n|$ converges, then show that $\sum a_n$ also converges. Is the converse true? Justify your answer.

ii. Show that $\sum \frac{1}{n^p}$ converges whenever $p > 1$.

B) Show that any monotonically increasing function on a finite interval $[a, b]$ is integrable. **(6)**

IV. A) Answer **<u>any one</u>** of the following. **(6)**

i. If $\boldsymbol{Q}$ is a refinement of any partition $\boldsymbol{P}$ of $[a, b]$ then show that $U(f, \boldsymbol{Q}) \leq U(f, \boldsymbol{P})$.

ii. Let $f: [a, b] \to \mathbb{R}$ be differentiable. If f' is integrable on $[a, b]$ show that $\int_a^b f'(x)\, dx = f(b) - f(a)$

B) Let f be continuous on $[a, b]$. Then show that there exists $c \in [a, b]$ such that **(6)**

$$\frac{1}{b-a}\int_a^b f(x)dx = f(c)$$

V. A) Answer **any one** of the following. (6)

i. Solve the following LPP using simplex method. Also check if an alternate solution exists.
Maximize, $Z = 4x_1 + 10x_2$
Subject to $2x_1 + x_2 \leq 50;$
$2x_1 + 5x_2 \leq 100;$
$2x_1 + 3x_2 \leq 90;$
$x_1 \geq 0, x_2 \geq 0$

ii. Solve the following LPP using duality.
Maximize, $Z = 15x_1 + 10x_2$
Subject to $3x_1 + 5x_2 \geq 5;$
$5x_1 + 2x_2 \geq 3;$
x_1 unrestricted, $x_2 \geq 0$

B) If X_0 be a feasible solution to the linear programming problem

Maximise $f(X) = CX$ subject to the conditions $AX \leq b, X \geq 0$ and W_0 be a feasible solution to the linear programming problem ; Minimise $g(W) = b^T W$ subject to the condition $A^T W \geq C^T, W \geq 0$

then prove that $CX_0 \leq b^T W_0$. (6)

VI. A) Answer **any one** of the following. (6)

i. Let $f_n : I \to \mathbb{R}$ be a sequence of functions with $f : I \to \mathbb{R}$ as its pointwise limit. Show that (f_n) converges uniformly to f if and only if the sequence $\sup\limits_{x \in I} |f_n(x) - f(x)|$ converges to 0.

ii. Discuss the nature of convergence of the following sequence of functions defined on $[0,1]$.

$$f_n(x) = x^2 e^{-nx}$$

B) A company uses annually 48000 units of a raw material costing ₹1.2 per unit. Placing each order cost ₹45 and the carrying cost is 15% per year of the average inventory. The replacement is instantaneous and no shortages allowed. Find the optimal size of the batch and the best time for the replenishment of the inventory. Also find the minimum average cost. (6)

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