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**CARMEL COLLEGE OF ARTS, SCIENCE & COMMERCE FOR WOMEN,
NUVEM-GOA**

SEMESTER END EXAMINATION, APRIL-MAY 2023

Sem. II of B.Sc **Class & Div: FYBSC (A&B)**

Course Title: MATRICES AND LINEAR ALGEBRA **Course Code: MTC 102**

Total marks: 80 **Date: 03/05/2023** **Duration: 2 hours** **Total No of pages: 3**

Instructions: *1. All questions are compulsory.*

2. Figures to the right indicate maximum marks to the questions.

3. Use of non-programmable calculator is allowed.

Q.1. Answer any FOUR of the following: **($4 \times 4 = 16$ marks)**

a. Find the Rank by reducing the matrix to its echelon form $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ 1 & 0 & 3 \\ 2 & -1 & 1 \end{bmatrix}$

b. Find A^{-1} by matrix method, where $A = \begin{bmatrix} 1 & 4 & 0 \\ -1 & 2 & 5 \\ 0 & 2 & 3 \end{bmatrix}$

c. Determine if the system has a non-trivial solution.

$$x + 2y + z = 0$$
$$y + 2z = 0$$
$$x + y - z = 0$$

d. State Cayley Hamilton theorem and verify it for $A = \begin{bmatrix} 1 & 2 \\ -3 & 9 \end{bmatrix}$

e. State the Cauchy-Schwarz inequality. Using Euclidean inner product on $\mathbb{R}^3$, show that $x = (-3,1,0)$ and $y = (2,-1,3)$ satisfy Cauchy-Schwarz inequality.

f. Write the definition of orthogonal and orthonormal basis for inner product space V

Q.2. Answer any FOUR of the following: **($4 \times 4 = 16$ marks)**

a. Let $V = \mathbb{R}^2$ be the set of all ordered pairs of real numbers. The operation of addition and scalar multiplication on $V = \mathbb{R}^2$ is defined as $(x_1, x_2) + (y_1, y_2) = (x_1 + y_2, x_2 + y_1)$ and $k(x_1, x_2) = (kx_1, kx_2)$ for all real scalars k. Show that V is not a vector space

b. Show that the set B= $\{(1,3,2), (1,3,0), (1,0,0)\}$ is a basis of vector space $\mathbb{R}^3$.

c. Let $\mathbb{R}^2$ have the Euclidean inner product.
Show that a parallelogram is a rhombus if and only if the diagonals are perpendicular to each other.

d. Define a line $l(p;d)$ in a vector space V, where $p \in V, d \in V$ and $d \neq 0$. Hence, show that $l(p;d) = l(p;\alpha d)$ for any $\alpha \in \mathbb{R} \setminus \{0\}$.

e. Show that the set $X = \{(x,y,z) \in \mathbb{R}^3 \mid x + 3y - z = 7\}$ is an affine space of $\mathbb{R}^3$.

f. Let $T: V \to V$ be a linear transformation then show that, T is one-one if and only if kernel of $T = \{0\}$

Q.3. A Answer any ONE of the following: **(6 marks)**

i Solve the system of equation

$$\begin{aligned} x + 2y + 3z &= 3 \\ 2x + 3y + 8z &= 4 \\ 3x + 2y + 17z &= 1 \end{aligned}$$

OR

ii Find λ and μ if the given system has i) Unique solution
ii) Infinite solutions
iii) No solution

$$\begin{aligned} 5x + 4y - 2z &= -3 \\ x - 13y + \lambda z &= 9 \\ 2x - 3y + 2z &= \mu \end{aligned}$$

Q.3. B. Answer the following: **(6 marks)**

Find the eigen values and eigen vectors of

$$A = \begin{bmatrix} 3 & 1 & 1 \\ -4 & -2 & -5 \\ 2 & 2 & 5 \end{bmatrix}.$$

Q.4. A Answer any ONE of the following: **(6 marks)**

i. Let V be a real inner product space. Show that
(i) $\|x + y\| \leq \|x\| + \|y\|$, for any $x, y \in V$
(ii) $|\|x\| - \|y\|| \leq \|x - y\|$, for any $x, y \in V$

OR

ii. Let $\mathbb{R}^3$ have the Euclidean inner product.
Use Gram -Schmidt orthogonalization process to convert basis $B = \{v_1, v_2, v_3\}$, where $v_1 = (1,1,1)$, $v_2 = (0,1,1)$, $v_3 = (0,0,1)$ into orthonormal basis.

Q4 B. Answer the following: **(6 marks)**

Diagonalise the matrix $A = \begin{bmatrix} 4 & 3 \\ 9 & -2 \end{bmatrix}$.

Q.5. A Answer any ONE of the following: **(6 marks)**

i. If W is an affine space in V, then show that W is a coset of some vector subspace W' of V.

OR

ii Let $S = \{(x, y, z) : x + y - z = 0\}$

i. Show that the set S is a subspace of $\mathbb{R}^3$.

ii. Find the basis of this subspace and hence determine the dimension of S.

iii. Extend the basis of S to the basis of $\mathbb{R}^3$.

Q.5. B. Answer the following: **(6 marks)**

State and prove necessary and the sufficient condition for a non-empty subset W of a vector space V to be a subspace of V.

Q.6. A Answer any ONE of the following: **(6 marks)**

i. $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $T(1,0,0) = (2,4,-1)$, $T(0,1,0) = (1,3,-2)$ and $T(0,0,1) = (0,-2,2)$. Find $T(x, y, z)$ and compute $T(-2,4,-1)$.

OR

ii. Let $T: V \to V'$ be a linear transformation, show that

i. Kernel of T is a subspace of V.

ii. Image of T is a subspace of V'.

Q.6. B. Answer the following: **(6 marks)**

Find the matrix of linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ given by $T(x, y, z) = (-x - y + z, x - 4y + z, 2x - 5y)$ with respect to the standard ordered basis $B = \{e_1, e_2, e_3\}$, where $e_1 = (1,0,0)$, $e_2 = (0,1,0)$, $e_3 = (0,0,1)$.

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