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B.Sc. (CBCS) (Semester -V)
EXAMINATION NOVEMBER 2022
Physics
Quantum Mechanics

[Duration : 2 Hours]

[Total Marks :80]

Instructions:

- 1) All questions are **compulsory**.
- 2) Figures to the **right** indicate **full** marks.
- 3) **Symbols** have their usual meaning unless otherwise stated.
- 4) Draw neat diagrams **wherever** necessary.
- 5) **Use** of calculator is **permitted**.
- 6) Given: $h=6.62 \times 10^{-34}$ J-s; $e=1.6 \times 10^{-19}$ C; $m_e=9.1 \times 10^{-31}$ Kg.

Q.1 Answer any **four** of the following:

[4x4=16]

- a) Illustrate Heisenberg uncertainty principle by Gamma ray thought experiment.
- b) State de-Broglie's hypothesis. Using the hypothesis, show that, an electron cannot exist in those orbits whose length is not a whole multiple of $h/2\pi$.
- c) Explain physical meaning of wave function $\psi(x, y, z, t)$. What is meant by Max Born's interpretation of wave function.
- d) What is the meaning of expectation value of a physical quantity? Explain how expectation values of momentum and energy of a particle can be determined using the wave function $\psi(x, t)$ in a one dimensional motion of the particle.
- e) Derive the formula expressing De-Broglie's wavelength (in Angstrom units) of an electron in terms of potential difference V (in volts) through which it is accelerated.
- f) Explain the significance of zero-point energy of a harmonic oscillator.

Q.2 Answer **any four** of the following:

[4x4=16]

- a) State Heisenberg Uncertainty principle. Show that, for a free particle the uncertainty principle can be written as $\Delta\lambda \cdot \Delta x \geq \frac{\lambda^2}{4\pi}$, where λ is the wavelength of de-Broglie wave associated with the particle.
- b) State the conditions that must be satisfied by a well behaved wave function.
- c) Deduce operator representation for kinetic energy and momentum of a material particle.
- d) What is a wave group description of a matter wave? Show that, the wave group velocity is equal to the particle velocity.

e) The wave function of a particle inside a one dimensional box of length L is

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \text{ calculate its mean position.}$$

f) Explain the phenomena of α -decay as tunneling across a potential barrier.

Q.3 A)

a) Show that the group velocity and phase velocities related by: $V_g = V_p - \lambda \frac{dV_p}{d\lambda}$ [3]

b) Explain in short the G. P. Thomson experiment as a qualitative confirmation of the de Broglie's hypothesis. [3]

OR

x) Explain single slit diffraction experiment in the light of Heisenberg's Uncertainty principle. [3]

y) Calculate the uncertainty in measurement of momentum of an electron if the uncertainty in locating it is 1\AA [3]

B) Describe Davisson and Germer experiment. How it does qualitatively and quantitatively confirms de Broglie's hypothesis? [6]

Q.4 A)

a) Show that: [3]

$$\left(\frac{d}{dx} + x\right)\left(\frac{d}{dx} - x\right) = \frac{d^2}{dx^2} - x^2 - 1$$

b) A particle limited to the x-axis has the wave function ' $\psi = ax'$ ' between ' $x=0$ ' and ' $x=1$ '; ' $\psi = 0$ ' elsewhere. Find probability the expectation value $\langle x \rangle$ of the particle's position. [3]

OR

x) An Eigen function of the operator $\frac{d^2}{dx^2}$ is $\psi = e^{4x}$ Find the corresponding eigen value. [3]

y) A hydrogen atom is 5.3×10^{-11} m. in radius. Estimate the minimum energy an electron can have in this atom. [3]

B) Obtain Schrödinger's time dependent equation using a free particle wave equation. [6]

Q.5 A)

- a) Draw diagram showing lowest three wave functions for a particle confined to a one dimensional harmonic oscillator. [3]
- b) Explain the phenomena of wave mechanical tunneling of potential barrier using a tunnel diode. [3]

OR

- x) Prove the following commutation relation:
 $[[A, B], C] + [[B, C], A] + [[C, A], B] = 0$ [3]
- y) Using energy and momentum operators, deduce Schrödinger's time dependent equation for a particle [3]

- B) A particle is incident on a step potential V_0 , at $x=0$ from the left with energy $E > V_0$. Find the expression for the reflection and transmission coefficients. [6]

Q.6 A)

- a) Sketch the Eigen function $\psi(x)$ and corresponding probability densities $\psi(x) * \psi(x)$ for the states $n=1, 2$ and 3 for a particle in a one dimensional box. [3]
- b) Calculate zero point energy of a harmonic oscillator of frequency 2×10^{15} htz. [3]

OR

- x) Electron with energy 1.0eV is incident on a barrier 10.0 eV high and 0.5 nm wide. Find the transmission probability: [3]
- y) Show that for a particle in a three dimensional box single value of energy, different quantum states are possible. Explain the term degeneracy. [3]

- B) Using Operator method, obtain general expression for Eigen functions of one dimensional harmonic oscillator. [6]