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B.SC (CBCS)(Semester-V)
EXAMINATION NOVEMBER2022
Mathematics
Combinatorics

[Time: 2 Hours] **[Max.Marks:80]**

Instructions:
1. All questions are compulsory ,however internal choice is available
2. Figures to the right indicate full marks
3. Use of scientific non-programmable calculator is allowed

Q.1 Attempt <u>any four</u> of the following **(4 x4=16 marks)**

a) State and prove the pigeon-hole principle
b) On the first day of the T.Y. B. Sc. Maths class, 30 students shook hands with each other exactly once. how many handshakes in total were there?
c) State and prove the strong induction principle
d) Is it true that c (n, n-1) =S (n, n-1)? justify
e) State the product formula and prove it, for ordinary generating functions
f) State and prove the exponential formula

Q.2 Attempt <u>any four</u> of the following **(4 x4=16 marks)**

a) For all positive integers n and k, show that the number of weak composition of n into k parts is
$\binom{n+k-1}{k-1}=\binom{n+k-1}{n}$

b) Find a formula for S(n,3) where S(n, k) are the sterling numbers of the second kind

c) Define a Ferrers shape "of a partition p=$(x_1, x_2 \ldots\ldots x_k)$when is a partition of n called self-conjugate Give an example of a self-conjugate partition

d) Let p:[n]→ [n]be a permutation and let x∈[n] then show that there exist a positive integer
$1 \leq i \leq n$ So that $p^i(x) = x$

e) Define
1. Sterling's numbers of first kind
2. Cycles is permutation

f) There are 14 students in a high school class who play soccer, and there are 17 students who play basketball four students play both games, how many students play at least one game?

Q.3 Attempt <u>any one</u> of the following **6 marks**

A. A committee of k people in to be chosen from a set of 7 women and 4 men ways are there to form the committee if

a) The committee consists of 3 women and 2 men only
b) The committee has 4 people and one of them must be Mr. Mathematics

OR

A. A manufacturing plant produces overs, At the last stage, an inspector marks the overs A (acceptable)or U (unacceptable), How many different sequences of IS, As and Us are possible in which the third U appears as the twelfth letter in the sequence

B. The genetic code of organism is stored in DNA molecular as a long string of four nucleotides: A (adenine),C(cytosine),G(guanine),and T (thymine).short strings of DNA can be "sequenced"- the sequence of letter determined –by various modern biotech methods although the DNA sequence for a single gene typically has hundreds or thousands of letters, there exist special enzymes that will split a long string into short fragment (which can be sequenced)by breaking the string immediately following each appearance of a particular letter. Suppose a C-enzyme (which splits after each appearance of C) breaks a 20-letter string into 8 fragments, which are identified to be: AC, AC, AAATC, C, C, C, TATA, TGGC. Note that each fragment, except the last one on the string must end with a C. How many different string could have given rise to this set of fragments?

6 marks

Q.4 Attempt <u>any one</u> of the following **6 marks**

A. For $n \epsilon \mathbb{N}$;show that

$$\binom{2(n+1)}{n+1} = \binom{2n}{n+1} + 2\binom{2n}{n} + \binom{2n}{n-1}$$

OR

A. Prove that for integers $0 \leq k \leq n-1$

$$\sum_{j=0}^{k} \binom{n}{j} = \sum_{j=0}^{k} \binom{n-1-j}{k-j} 2^j$$

B. What is the coefficient of x^k in $(1-x)^{-n}$?justify

6 marks

Q.5 Attempt <u>any one</u> of the following **6 marks**

A. For all positive integer $k \leq n$,prove that $S(n_1 k) = S(n-1, k-1) + k.S(n-1, k)$

OR

A. For all real numbers x, and all non-negative integers Show that $x^n =$

$\sum_{k=0}^{n} s(n, k)(x)_k$

B. State and prove the sieve formula (also known as the Inclusion-Exclusion principle)

6 marks

Q.6 Attempt <u>any one</u> of the following **6 marks**

A. The frog population of a lake groups found fold each on the first day of each year,100 frogs are taken out of the lake and shipped into another lake. Assuming that there were so frogs in the lake originally how many frogs will be in the lake in 20 years?

OR

A. A Semester at a technical university consists of n days.at the beginning of each semester, the Dean of engineering design the term by splitting it into two parts and choosing some days for independent study in both parts of the semester. In how many different ways can she plan the semester with their constraint?

B. Let $a_0 = 1$ and let $a_{n+1} = (n+1)(a_n - n + 1)$ if $n \geq 0$ find a closed formula for a_n. **6 marks**