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B.Sc. (CBCS) (Semester -V)
EXAMINATION NOVEMBER 2022
Physics
Mathematical Physics & Electromagnetic Theory I

[Duration : 2 Hours] **[Total Marks : 80]**

Instructions:
1) **All** questions are **compulsory**.
2) Figures to the **right** indicate full marks.
3) Symbols have their **usual** meaning, unless otherwise stated.
4) Draw illustrated diagrams **wherever** necessary.
5) Use of log tables and calculators is allowed.

Constants:
Permittivity of free space $\varepsilon_0 = 8.854 \times 10^{-12}\ C^2/\ Nm^2$

Q.1 Answer **any four** of the following: **(4x4=16)**

a) Show that $\vec{\nabla}\Phi$ is a vector perpendicular to the surface Φ (x, y, z) = c where c is a constant.
b) Show that $\vec{\nabla}.(\vec{\nabla} \times \vec{A}) = 0$
c) Test the following equation for exactness and solve if it is exact:
$$e^x sinydx + (e^x + 1)cosydy = 0$$
d) State & prove second uniqueness theorem to the solution of Laplace's equation.
e) i) State Coulomb's law in electrostatics.
ii) Electric field inside the conductor is zero. Explain.
f) Obtain the relationship between electric susceptibility and dielectric constant.

Q.2 Answer **any four** of the following: **(4x4=16)**

a) Calculate the work done by the force $\vec{F} = 4y\hat{\imath} + xy\hat{\jmath}$ N in moving an object along a straight line from A(0,0,0) to B(2,1,0) meters.
b) Define gamma function and beta function. Give the equation stating the relation between gamma and beta function.
c) The reduced mass μ of a system of two masses m_1 and m_2 is defined by $\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$. If m_1 is increased by 1% what fractional change in m_2 leaves μ unchanged?
d) An infinite plane carries a uniform surface charge σ. Find its electric field.
e) Using the concept of molecular field obtain the condition for permanent polarization.
f) Calculate the work done to move a charge of 2μc from infinity to point at a distance 4cm from 3μc charge.

Q.3 A) Prove the following relation: **(6)**
$$\iiint_\tau \vec{\nabla} \times \vec{A}\, d\tau = \oiint_\sigma \hat{n} \times \vec{A}\, d\sigma$$
Hence solve the following:

If σ is a closed surface which encloses a volume τ. Prove that

$$-\oiint_{\sigma} \vec{r} \times \overrightarrow{d\sigma} = \iiint_{\tau} \vec{\nabla} \times \vec{r}\, d\tau$$

OR

A) If a vector field is given by $\vec{F}(x^2 - y^2 + x)\hat{\imath} - (2xy + y)\hat{\jmath}$ is this field irrotational? If so find its scalar potential. **(6)**

B) Prove $\nabla^2 r^n = n(n+1)r^{(n-2)}$ **(6)**

Q.4 A) Solve the equation **(6)**

$$\frac{\partial^2 z(x,y)}{\partial x \partial y} = x^2 y$$

by using method of direct integration subject to conditions $z(x,0) = x^2$ and $z(1,y) = cosy$

OR

A) Solve Laplace's two-dimensional equation in Cartesian co-ordinates by using method of separation of variables. **(6)**

B) Expand $f(x) = x\ for - \pi \leq x \leq \pi$ in a Fourier series. **(6)**

Q.5 A) What is an electric dipole? Obtain an expression for electric field due to a dipole. **(6)**

OR

A) A point charge q is kept near infinite grounded conducting plane. Use method of images to find the expression for electric field, surface charge density and induced charge. **(6)**

B) Find the solution of Laplace's equation in spherical coordinates using zonal harmonics assuming potential to be independent of azimuthal angle. **(6)**

Q.6 A) Show that polarization of dielectric medium give rise to volume density of charge $\rho_p = -\vec{\nabla}.\vec{P}$ and surface charge density $\sigma_p = \vec{P}.\hat{n}$. **(6)**

OR

A) Show that molecular field in a dielectric is given by $E_m = E + \frac{P}{3\varepsilon_0}$ where P is polarization. **(6)**

B) Show that the electrostatic potential energy of a group of m point charges is given by **(6)**

$$W = \sum_{j=1}^{m} q_j U_j$$