16-11-22

Paper / Subject Code: MTC107 / Mathematics - Calculus of 2 & 3 variable

MTC 107

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B. SC. (Semester- V)
EXAMINATION NOVEMBER 2022
MATHEMATICS
Calculus of 2 & 3 variable

[Duration : 3 Hours] **[Mix Marks : 120]**

Please check whether you have got the right question paper.

Instructions:
1. All questions are compulsory. However internal choice is available.
2. Figures to the right indicate full marks to each question/ Sub question
3. Use of scientific / Non-programmable calculators is allowed.

I. Answer any five of the following: - **5 x 4=20**

a. The equation of motion of a particle P of mass m is given by $m\left(\frac{d^2r}{dt^2}\right) = f(r)\hat{r}, where\ \bar{r}$ is the position vector of P measured from an origin $O. \hat{r}$ is a unit vector in the direction of $\bar{r}$ and $f(r)$ is a function of the distance of P from Q. $show\ that\ \bar{r}x\frac{d\bar{r}}{dt}$= C, where C is a constant vector.

b. $\bar{A} = u\bar{\imath} + u^2\bar{\jmath} + u^3\bar{k}$
$\bar{B} = u^3\bar{\imath} + u^2\bar{\jmath} + u\bar{k}$
Find $\frac{d}{du}\ (\bar{A}\ \times\ \bar{B})$

c. Find the directional derivative of the function $x^2 + y^2 + z^2\ at\ (3,6,9)$ in the direction whose direction cosines are ${}^{1}/{}_{3}\ ,{}^{2}/{}_{3}, {}^{2}/{}_{3}.$

d. If $\bar{F} = \nabla(x^3 + y^3 + z^3 - 3xyz$
Find $\nabla\ \times\ \bar{F}$

e. $f(x, y) = \frac{\cos x + e^{xy}}{x^2+y^2}$
$show\ that\ f$ is differentiable at all points $(d, y) \neq (0,0)$

f. Find a unit normal to the surface $\sin(xy) = e^x\ at\ (1, {}^{\pi}/{}_{2}, 0)$

g. Use $(\varepsilon - 0)dy^n$to show that $f(x, y) = \frac{x^3-y^3}{x^2+y^2}, (x, y) \neq (0,0)$

$= 0, \quad (x, y) = (0,0)$

is continuous at (0,0)

II. **Answer <u>any five</u> of the following** **5 x 4=20**

a. Compute the jacobian determinant of
$x = u + \vartheta$
$y = u - \vartheta$

b. Evaluate $\int_c y\,dx + x\,dy$ if c is any pat joining (0,0) and (1,1)

c. Define Parameterized surface

d. $C : C_1(t) = (t, 0{,}0)$ is a line segment from (0,0,0) to (1,0,0)

$C: C_2\,(t) = (1 - t, 0{,}0)$ is a line segment joining (0,0,0) to (1, 0, 0). Let $f(x, y, z) = 1$
Evaluate the line integral of F along C_1 and along C_2 are the two values equal. Explain.

e. State the sunder mental theorem of calculus

f. Define the moment of Inertia about the Z-axis.

g. Compute $\iint_s xd\ \ s$ where S is the triangle with vectors (1,0,0), (0,1,0), (0,0,1)

III. **A. Answer <u>any one</u> of the following**

a. i. For what values of t is $f(t) = (sint)\bar{\imath} + (1 - t)^{-1}\bar{\jmath} + (\ln t)\,\bar{k}$ continuous ? [3]
ii. Show that $f(x, y) = x^2y + xy^3$ is differentiable for all (x, y) [4]
iii. $F(t) = t\bar{\imath} + \sin t\bar{\jmath} + cost\bar{k}\ C_1(t) = t\bar{\imath} + (1 - t)\bar{\jmath} + t^2\bar{k}$. Find $\frac{d}{dt}\ (F.G)$ [3]

OR

b. i. For what values of is $C_1\,(t) = 1t1\bar{\imath} + \cos t\,\bar{\jmath} + (t - 5)\bar{k}$ differentiable ? [3]
ii. Let $Z = \sqrt{x^2 + 2xy,}$ where $x = cos\theta\,, y = \sin\theta\ find\ \frac{dz}{d\theta}$ [3]
iii. if y is a differentiable function of x such that $Sin\,(x + y) + cos(x - y) = y\ find \frac{dy}{dx}$ [4]

B. show that the function $f(x, y) = \frac{x^3 - y^3}{x^2 + y^2}, (x, y) \neq (0{,}0)$ [10]
$$= 0, \quad (x, y) = (0{,}0)$$

Is continuous at (0,0) possesses $f_x(0{,}0)$ and $f_y(0{,}0)$ but is not differentiable at (0,0)

IV. A. Answer **any one** of the following: -

a. What is the volume of the largest rectangular parallelepiped which can be inscribed in the ellipsoid $\frac{x^2}{9}+\frac{y^2}{16}+\frac{z^2}{36}=1$ **[10]**

OR

b. i. Investigate the maxima and minima of the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20$ **[5]**

ii. Examine for the change in the order of derivation at the origin for the function: $(i.e\ f_{xy}\ (0,0), f_{yx}(0,0)\ f(x,y) = |x^2 - y^2|$ **[5]**

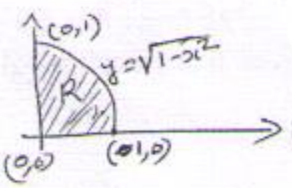
B. i. Parametrize the curve $x = t, y = t^2, z = t^3$ in terms of arc length. **[4]**

ii. Find $\nabla.(r^n\bar{r})$ and $\nabla\, x\,(r^n\bar{r})$ **[6]**

V. A. Answer **any one** of the following:

a) i. Evaluate $\iint_R \sqrt{1-y^2}\, dydx$ where R is the region **[5]**

as in fig:



ii. Evaluate $\iint x^2y\, dxdy$ taken over the positive quadrant of the circle $x^2 + y^2 = a^2$, by changing to Polar coordinates. **[5]**

OR

b) i. Evaluate $\iint (x^2 + y^2)\ dx\, dy$ over the region $D: \{(x,y)\ |x \geq 0, y \geq 0, x + y\ \leq 1\}$ **[5]**

ii. Find the mass center of the first Quadrant area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, using double integrds. **[5]**

B. i. Evaluate $\iiint \frac{1}{\sqrt{1-x^2-y^2-z^2}}\, dxdydz$ over the region bounded by the Sphere $x^2 + y^2 + z^2 = 1$ **[5]**

[using spherical polar coordinates]

ii. find the Average value of f over the Triangle with vertices (0,0), (0,1) and (1,0) where $f(x,y) = e^{x+y}$ **[5]**

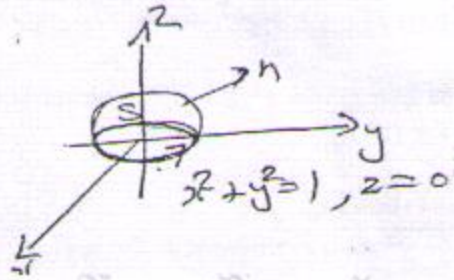
VI. A. Answer **any one** of the following:

a) i. Evaluate $\iint_s A.n\ ds$, where $A = z\bar{i} + x\bar{j} - 3y^2z\bar{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between z=0 and z=5. **[5]**

ii. suppose f is the force field $f(x, y, z) = x^3\bar{\imath} + y\bar{\jmath} + 2z\bar{k}$. Find the work done by f along the circle of radius Q in the yz plane. [5]

OR

$\rightarrow$ b) i) Use stoke's theorem to evaluate $\iint_S (\nabla x f).ds$ where S is the portion of the Sphere of radius 2. $F = y\bar{\imath} - x\bar{\jmath} + e^{xz-} K$. As shown. [5]



ii) If $A = (4xy - 3x^2z^2)\bar{\imath} + 2x^2\bar{\jmath} - 2x^3 2\bar{k}$ Prove that $\int_c A - dr$ is independent of the curve C joining two given points. Also find ϕ such that $A = \nabla\phi$ [5]

B. State and prove the Gauss divergence theorem. [10]