MTE-101

Total No. of Printed Pages:03

B. Sc. (CBCS) (Semester-V)
EXAMINATION NOVEMBER 2022
Mathematics
Foundations of Mathematics

[Duration :2 Hours] **[Total Marks :80]**

Instructions:
1. All questions are compulsory, however internal choices are available.
2. Figures to the right indicate full marks.
3. Use of scientific non-programmable calculator is allowed.

Q.1 Answer any four of the following:- **(16)**

a. When is a subset of $\mathbb{R}$ said to be bounded below?
Write your answer and its negation using quantifiers.

b. Write converse, inverse and contrapositive of the following statement: -
For x,y in $\mathbb{R}$ if xy is rational , then x is rational and y is rational

c. Identify the sets $A = \{x \epsilon \mathbb{R}; x^2 \geq 4\}$ and $B = \{x \epsilon \mathbb{R}; 2 <|x-1| < 5\}$.
Hence find $A \cap B$.

d. Prove or disprove the following:-
For any sets A, B, C and D
$(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)$

e. Express the closed interval [0, 1] in $\mathbb{R}$ as intersection of a family of open intervals indexed by $n \epsilon \mathbb{N}$. Justify your answer.

f. Give examples of functions f , g $\mathbb{Z} \to \mathbb{Z}$ such that
i) g.f is injective but g is not.
ii) g.f is surjective but f is not.

Q.2 Answer any four of the following:- **(16)**

a. k is a fixed positive integer and $a \equiv b \ (mod\ k)$.
Using induction principle , show that $a^n \equiv b^n (mod\ k)\ \ for\ n \epsilon \mathbb{N}$

b. Prove that every amount of postage that is at least 12 rupees can be made from 4-rupee and 5-rupee stamps

c. $For\ n \epsilon \mathbb{N}$, let in denote the set {1,2…n}
If $m, n \in \mathbb{N}\ and\ m < n, show\ that\ there\ is\ no\ onto\ map\ from\ I_m to\ I_n$

d. If A, B are countable, show that $A \times B$ is countable.

e. Let $(X, \leq)$ be a partially ordered set. Define a relation R on X as follows:-
$x\, Ry$ *if and only if* $y \leq x$.
Is R a partial ordered on x? Justify

f. Define the terms 'Maximum element' and 'Minimum element' of a subset A of a partially ordered set $(x, \leq)$
If maximum element of A exists, show that it is unique.

Q.3 A. Answer any one of the following:- **(6)**

a. Prove or disprove the following:-
i) For $x \in \mathbb{Z}, if\ 4x^2 - 4x + 5\ is\ odd, then\ x\ is\ odd$
ii) For any $x \in [0, \pi/2, Sinx + Cosx \geq 1$

b. If $\{F_\alpha . \alpha \in \wedge\}$ is a family of subsets of a set X indexed by $\wedge\ and$ A is any subset of X, show that
i) $(\cap_{\alpha\epsilon\wedge} F_\alpha)^c = \cup_{\alpha \in \wedge} F_\alpha^c)$
and ii) $A \cup (\cap_{\alpha\epsilon\wedge} F_\alpha) = \cap_{\alpha\epsilon\wedge}(A \cup F_\alpha)$
Here B^c denotes the complement of B

B. If $A = \{x \in \mathbb{R} ; x (x-1)(x-2) < 0 \}$ and **(6)**
$B = \{x \epsilon \mathbb{R} ; x^2 - 5x - 6 \geq 0\}$,
$Find\ A \cap B\ and\ A \cup B.$

Q.4 A. Answer any one of the following:- **(6)**

a. Show that a function $f : X \to Y\ is\ surjective\ if\ and\ only\ if$
$Y \setminus f(A) \subseteq f(X \setminus A) for\ all\ subset\ A\ of\ X$

b. Show that $f : [0,1] \to [0,1] given\ by\ f(x) = \frac{1-x}{1+x}\ is\ a\ bijection$
Find f^{-1}

B. ~ be an equivalence relation on a set X and for $x \in X$, let [x] denote the equivalence class of x **(6)**
Prove the following:-
i) $x \epsilon [x] \quad for\ x \epsilon X$
ii) $x \epsilon [y] \quad if\ and\ only\ [x] = [y] for\ x, y \epsilon X$
iii) For $x, y \epsilon X,\ [x] = [y]\ or\ [x] \cap [y] = \phi$

Q.5 A. Answer any one of the following:- **(6)**

a. State i) The well – ordering principle and
ii) Division algorithm for the set $\mathbb{N}$ of natural numbers.
Using (i), prove (ii)

b. Use 'The well – ordering principle' to prove that for any natural number n ,
$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

B. Show that for any non-empty set A , the following are equivalent:- **(6)**
i) A is countable
ii) There is an injective map from A to $\mathbb{N}$
iii) There is a surjective map from $\mathbb{N}$ to A

Q.6 A. Answer any one of the following:- **(6)**

a. Let $(X, \leq)$ and $(Y, \leq)$ be partially ordered sets.
Define R on $(X \times Y)$ by
$(x_1, y_1)\, R\, (x_2, y_2)$ iff $either$ $x_1 < x_2$ or $(x_1 = x_2$ and $y_1 \leq y_2)$
Show that R is a partial order on $(X \times Y)$

b. Let $\leq$ be a relation on $\mathbb{N}$ defined as $m \leq n$ if and only m divides n and
A = {1, 2, 3, 4, 5, 6, 9, 15, 18, 24, 36}
Show that $\leq$ is a partial order on $\mathbb{N}$
Draw the Hasse diagram for A
Find i) Maximum of A
ii) Maximal elements of A
iii) Lub of A

B a) Let A be a subset of partially ordered set $(X, \leq)$ **(6)**
Define the following terms:
i) Maximal and Minimal elements of A
ii) LUB and GLB of A
If LUB of A exists, show that it unique.
b) Let $X = P(\mathbb{R}) \setminus \{\phi , \mathbb{R}\}$ being ordered by inclusion.
Identify maximal and minimal elements of X.
$P(\mathbb{R})$ denotes the power set of $(\mathbb{R})$.