

CARMEL COLLEGE OF ARTS, SCIENCE & COMMERCE FOR WOMEN,
NUVEM-GOA

SEMESTER END EXAMINATION, November 2022

Semester: III Of BSC

Course Title: Ordinary Differential equations and Discrete Mathematics

Course Code: MTC103

Total marks: 80

Date: 26/11/2022 Duration: 2 hrs Total No of pages : 3

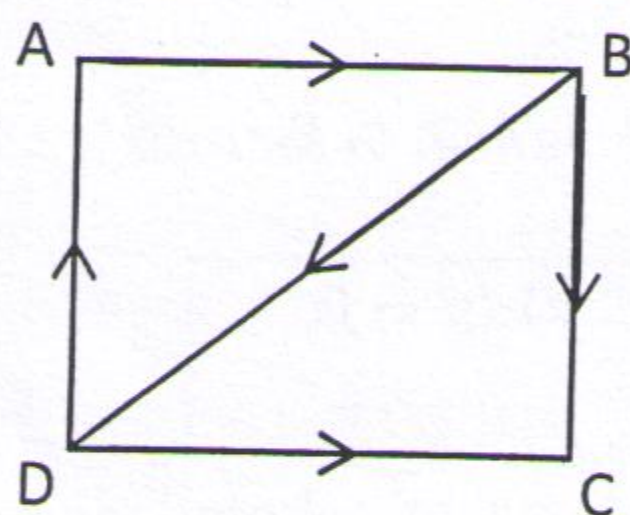
- Instructions:
1. All questions are compulsory
 2. Figures to the right indicate marks
 3. Use of non-programmable calculator is allowed.

Q.1. Answer any 4 of the following questions:

(16)

1. Solve $\frac{dy}{dx} = \frac{x+y-3}{x+y+3}$

2. Determine the number of vertices and edges and find the in-degree and out-degree of each vertex for the given directed multigraph.



3. Show that $x(t) = 3t - 2$, $y(t) = -2t + 3$ is the solution of the system

$$\frac{dx}{dt} = x + 2y + t - 1$$

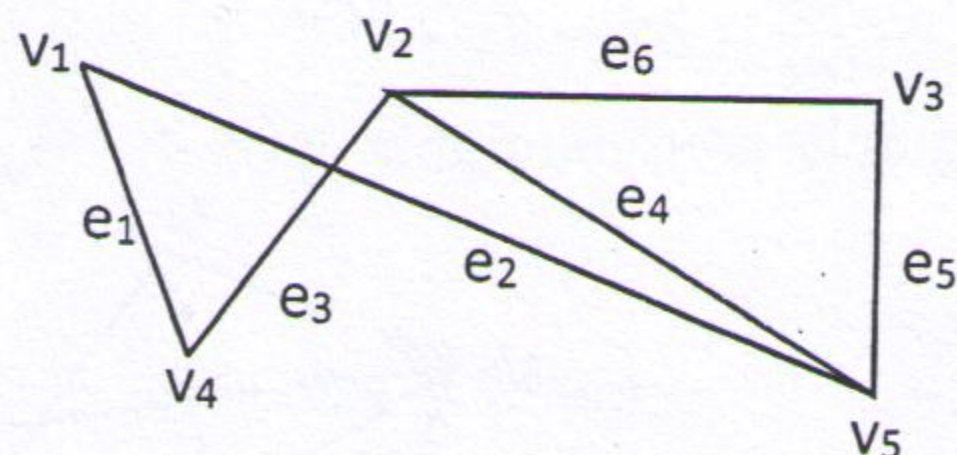
$$\frac{dy}{dt} = 3x + 2y - 5t - 2$$

4. Convert the differential equation to an equivalent system of first order equations

$$y''' + 2y'' - y' + 3y = x.$$

5. State the necessary and sufficient condition for the existence of
- a. Eulerian path in a graph.
 - b. Eulerian circuit in a graph

6. Represent the given graph with an incidence matrix

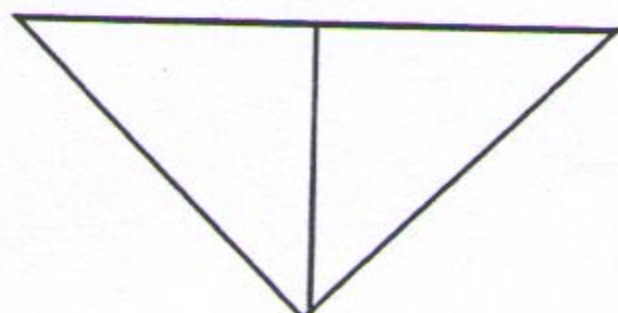


Q.2. Answer any 4 of the following questions:

(16)

1. Show that $y(x) = ax + bx^2$ is the general solution of $x^2y'' - 2xy' + 2y = 0$ on any interval not containing the origin. Find the particular solution given $y(1) = 3$, $y'(1) = 5$

2. Find the general solution of $y'' - 2y' + 5y = 25x^2 + 12$, by using method of undetermined coefficients.
3. Solve $(D - 1)^2 y = e^{2x} - e^{-2x}$ using D-operator method.
4. If $y_1(x)$ and $y_2(x)$ are two solutions of $\frac{d^2 y}{dx^2} + P(x) \frac{dy}{dx} + Q(x)y = 0$. Then prove that $c_1 y_1 + c_2 y_2$ is also a solution, where c_1, c_2 are arbitrary constants.
5. Define the terms: 1. transport network. 2. Prefix codes.
6. Find all the spanning trees of the given graph.



Q.3. Answer the following questions:

A. Answer any one of the following questions:

a. Solve $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$.

(6)

OR

b. Solve $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$.

B. If $W(t)$ is the Wronskian of two solutions of a homogenous system of equations (6)

$$\frac{dx}{dt} = a_1x + b_1y$$

$$\frac{dy}{dt} = a_2x + b_2y$$

then, show that $W(t)$ is either identically equal to zero or nowhere zero.

Q.4. Answer the following questions:

A. Answer any one of the following questions:

(6)

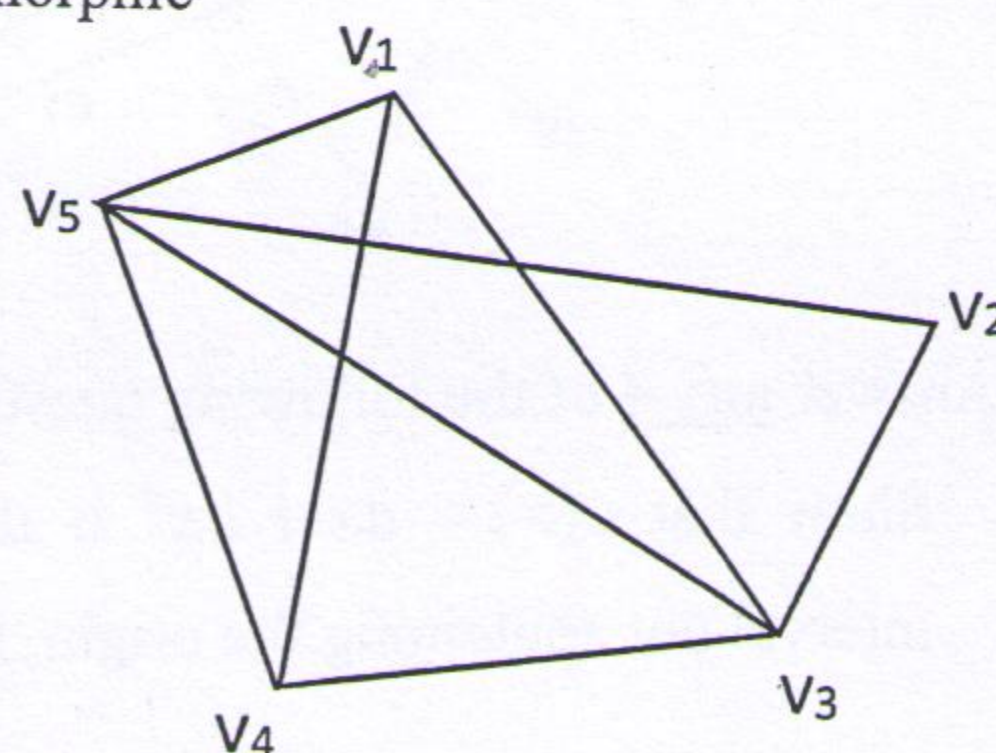
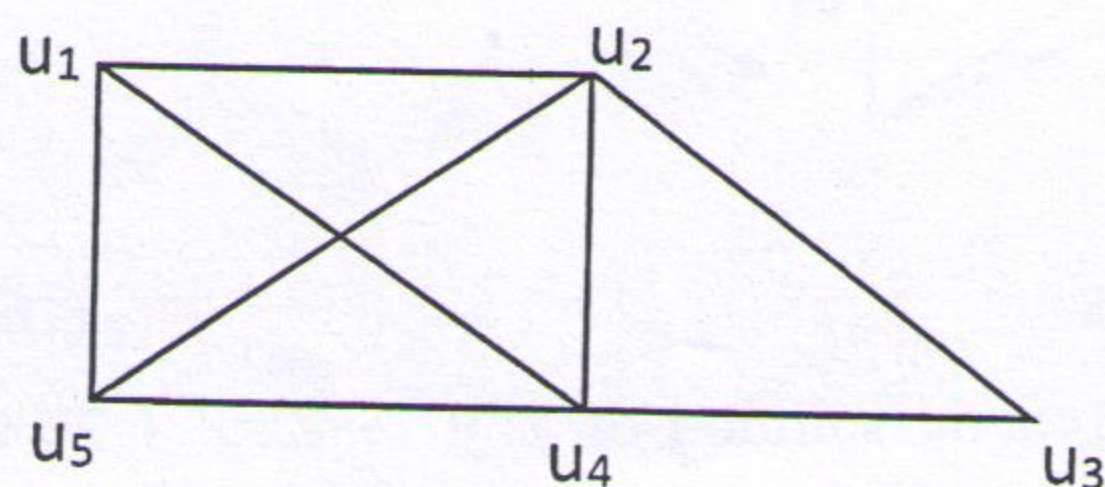
a. Solve the system of differential equations $\frac{dx}{dt} = 3x - 4y$
 $\frac{dy}{dt} = x - y$

OR

b. Solve the system of equations $\frac{dx}{dt} = 2x - y + e^t$
 $\frac{dy}{dt} = 3x - 2y + e^{-t}$

B. Determine whether the given graphs are isomorphic

(6)



Q.5. Answer the following questions:

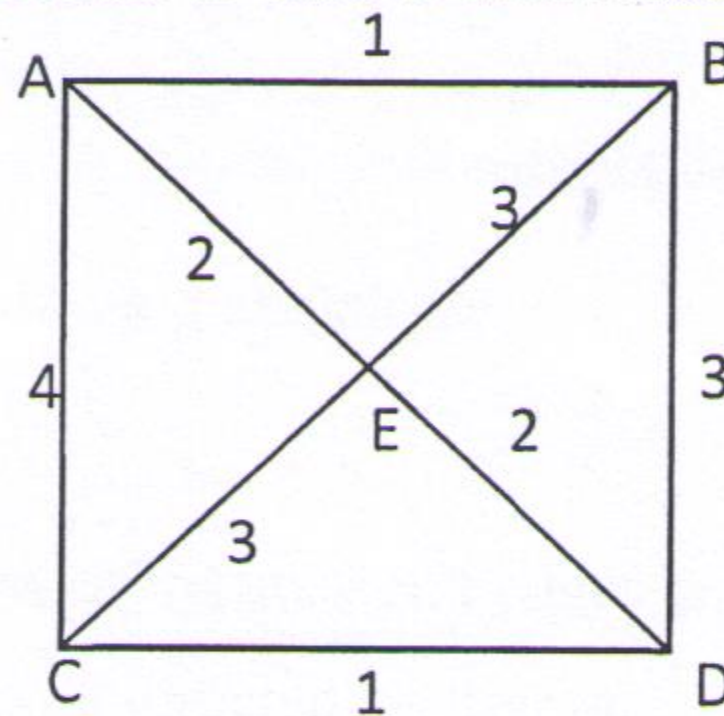
A. Answer any one of the following questions:

(6)

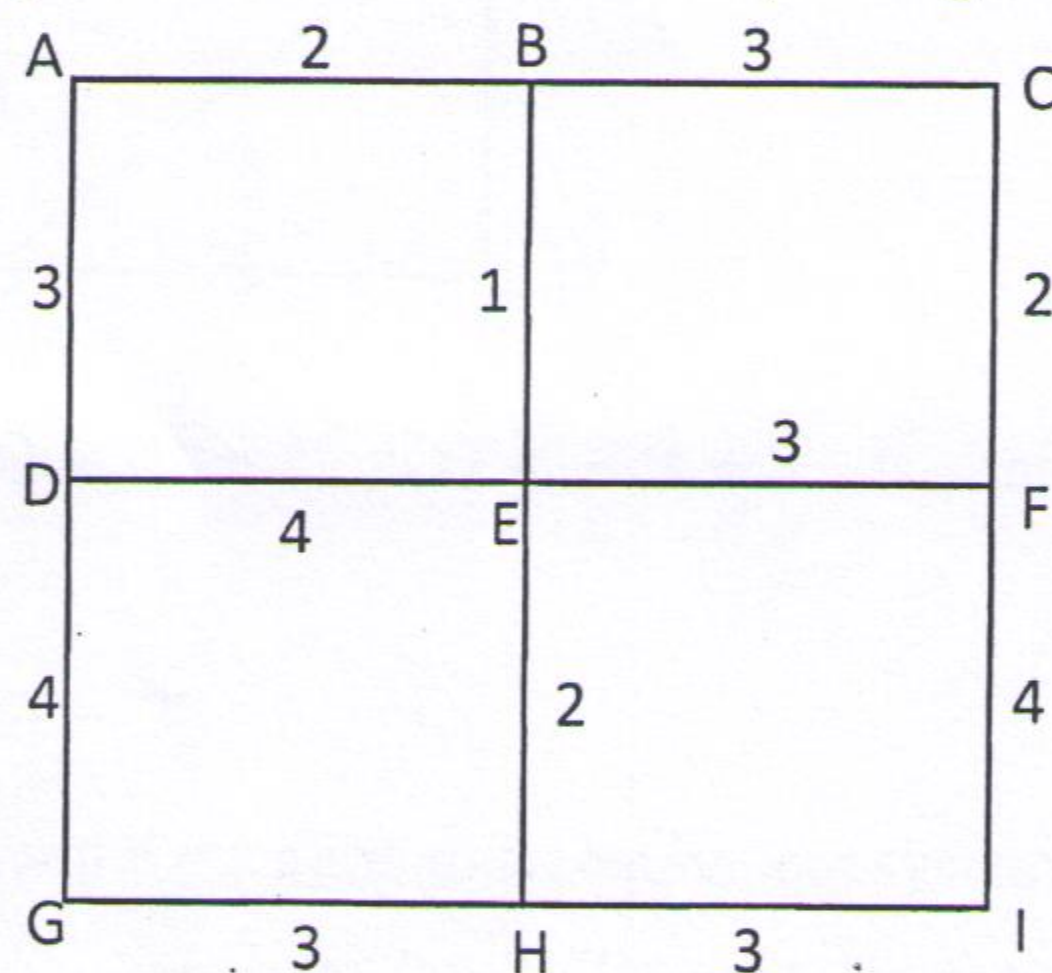
a. Solve the differential equation $y'' + 2y' + y = e^{-x} \log x$

OR

b. Use Prim's algorithm to find a minimum spanning tree for the given weighted graph.



B. Use Kruskal's algorithm to find a minimum spanning tree for the given weighted graph.



(6)

Q.6. Answer the following questions:

A. Answer any one of the following questions:

(6)

a. Solve $(D^2 - 5D + 6)y = e^x \cos 2x + e^{-x}$ using D-operator method

OR

b. Solve $(D^3 + 8)y = x^4 + 2x + 1 + e^{-2x}$ using D-operator method.

B. Solve $x^2 y'' - xy' - 3y = x^2 \log x$.

(6)