CARMEL COLLEGE OF ARTS, SCIENCE AND COMMERCE FOR WOMEN, NUVEM-GOA

SEMESTER END EXAMINATION, NOVEMBER, 2022

Semester: I OF B.Sc

Course Title : CALCULUS AND NUMERICAL METHODS **Course Code: MTC 101**

Total marks: 80 **Date:** 16|11|2022 **Duration: 2 hours** **Total No of pages: 2**

Instructions: *1. All questions are compulsory.*

2. Figures to the right indicate maximum marks to the questions.

3. Use of non-programmable calculators is allowed.

Q.1. Answer any FOUR of the following: **($4\times4 = 16$ marks)**

a. For $x,\ y \in \mathbb{R}$, show that $|x| \leq y$ if and only if $-y \leq x \leq y$

b. For $x,\ y \in \mathbb{R}$, show that if $xy = 0$ then either $x = 0$ or $y = 0$

c. If $A \subset B$ and B is bounded, show that A is bounded.

d. For $x,\ y \in \mathbb{R}$, define ' x *is less than* y'
Prove that $x < y$ and $z < w$ implies $x + z < y + w$

e. Prove that, "Every convergent sequence is bounded"

f. Show that, the sequence $\langle x_n \rangle$ defined as $x_n = \frac{105n^2-8}{3n^2+91n-3}$ converges, state the results used.

Q.2. Answer any FOUR of the following: **($4\times4 = 16$ marks)**

a. Discuss the continuity and differentiability of the function at $x = 2$

$$f(x) = \begin{cases} 2x - 3 & if\ 0 \leq x \leq 2 \\ x^2 - 3 & if\ 2 < x \leq 4 \end{cases}$$

b. Find the interval in which the function $f: \mathbb{R} \to \mathbb{R}$ given by
$f(x) = 2x^3 - 12x^2 + 18x + 81$ is increasing.

c. Verify the Rolle's theorem for the following function
$f(x) = x^2 - 6x + 8$ on $[2,4]$

d. Evaluate $\lim_{x\to1} \frac{1}{x-1}\left(\frac{1}{x+3} - \frac{2}{3x+5}\right)$

e. Construct Forward Difference table for the following data.

X	2	2.5	3	3.5	4
Y	1.39	12.18	20.08	38.12	54.6

f. i) If $h = 1$ then find $\Delta^4 (x+1)(x+2)(2x+3)(2-4x)$ using fundamental theorem of difference calculus.

ii) When Mrs. Dessai got home, she found that the exact weight of the 2 kg tomatoes she had purchased was 1.7 kg. Find the Relative error, Absolute error and Percentage error.

Q.3. A Answer any ONE of the following: **(6 marks)**

i If A and B are bounded subsets of $\mathbb{R}$, Show that $A \cup B$ is bounded in $\mathbb{R}$.

OR

ii For real number x define the absolute value, hence show that for all real numbers x and y, $|x+y| \leq |x| + |y|$

Q.3. B. Answer the following: **(6 marks)**

Prove that if a function f is continuous on $[a, b]$ and $f(a) \neq f(b)$ then it assumes every value between $f(a)$ and $f(b)$.

Q.4. A Answer any ONE of the following: **(6 marks)**

i. Prove that: If $\langle x_n \rangle$ and $\langle y_n \rangle$ are two sequences such that $x_n \to x$ and $y_n \to y$, then $x_n + y_n \to x + y$.

OR

ii. Verify that the sequence $\langle x_n \rangle$ defined by $x_1 = 1$ $x_{n+1} = \sqrt{2x_n}$, for $n > 1$ is monotone and bounded. What can you state about its convergence?

Q4 B. Answer the following: **(6 marks)**

Prove that 'Every Subsequence of a convergent sequence in $\mathbb{R}$ converges to the same limit as the sequence.'

Q.5. A Answer any ONE of the following: **(6 marks)**

i. Using Newton Cotes Quadrature formula, derive Simpson's 1/3rd rule for the data points (x_i, y_i) $i = 1,2,3 \ldots . n$.

OR

ii Using usual notations, prove that

a. $\Delta E = E\Delta$ b. $\Delta = \nabla E$ c. $\nabla = 1 - E^{-1}$

Q.5. B. Answer the following: **(6 marks)**

Prove that, if a function is continuous in a closed interval, then it is bounded.

Q.6. A Answer any ONE of the following: **(6 marks)**

i. Use the Taylor's theorem to write the polynomial $f(x) = 2x^5 + x^2 - 3x - 5$ in powers of $(x+1)$.

OR

ii. If $f: \mathbb{R} \to \mathbb{R}$ is differentiable at c in $\mathbb{R}$ and f has a maximum at c then show that $f'(c) = 0$

Q.6. B. Answer the following: **(6 marks)**

State and prove Lagrange's Mean Value Theorem.

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