CARMEL COLLEGE OF ARTS, SCIENCE & COMMERCE FOR WOMEN, NUVEM-GOA

SEMESTER END EXAMINATION, JUNE 2022

Semester: IV OF BSC

Course Title: Analysis and Operations Research Course Code: UMTC 104

Total marks:80 Date:16.06.2022 Duration: 2 hours Total No of pages: 02

Instructions: *i) All questions are compulsory*
ii) Figures to the right indicate full marks
iii) Use of non programmable calculators is allowed.

I. Answer <u>any four</u> of the following. (4x4 = 16)

1. Define the following terms.
   i Sequence of Partial Sums of a sequence
   ii Convergent Series
2. Show that if $\sum a_n$ converges, then the sequence (a_n) converges to 0.
3. Let $f: [0,3] \to \mathbb{R}$ defined as
$$f(x) = \begin{cases} 1 & if \quad x \in [0,1) \\ 3 & if \quad x \in [1,2) \\ 2 & if \quad x \in [2,3] \end{cases}$$
   If $\epsilon > 0$ is any arbitrary number, find a partition P such that
$$U(P,f) - L(P,f) < \epsilon$$
4. Evaluate $\lim_{h\to 0} \frac{1}{h} \int_3^{3+h} e^{t^2} dt$
5. Check whether the following series converge. Justify your answers.
   i. $\sum \frac{2^n}{n^n}$
   ii. $\sum \frac{\log(n)}{n^2}$
6. Show that the series $\sum \frac{n^2}{n^3-2}$ diverges.

II. Answer <u>any four</u> of the following. (4x4 = 16)

1. Define the following terms.
   i Pointwise limit of a sequence of Real valued functions over a subset of $\mathbb{R}$.
   ii Uniformly convergent sequence of functions.
2. Find the limit of the sequence (f_n) where $f_n(x) = x^n$ for $x \in (0,1]$. Is the convergence uniform?
3. Check whether the series $\sum f_n$ converges uniformly over [0,1], where $f_n(x) = \frac{\sin(nx)}{n^n 2^n}$.
4. For each $n \in \mathbb{N}$, let $P_n = \{0, \frac{1}{n}, \frac{2}{n}, \ldots 1\}$ be a partition of [0,1]. Calculate $\lim_{n\to\infty} \mathrm{U}(f, P_n)$ for the function $f(x) = x^2$.
5. A company has three factories & three customers. The company furnishes the following schedule of loss per unit on transportation of its goods to the customers in Rupees.

Factory	Customers			
	A	B	C	Supply
P	4	19	22	50
Q	2	9	14	30
R	6	6	16	70
Demand	70	20	60	

Find initial basic feasible solution using North West Corner method.

6. Solve the following LPP using Graphical Method
   Minimize $Z = 5x + 7y$
   Subject to $3x - 7y \leq -14$
   $x + y \leq 4$;
   $5x - 2y \geq 25$;
   $x \geq 0, y \geq 0$

III. A) Answer <u>any one</u> of the following. **(6)**

i Solve the following LPP using Simplex Method. Also check if an alternate solution exists.
Maximize $Z = 7x_1 + 5x_2$
Subject to
$x_1 + 2x_2 \leq 6$
$4x_1 + 3x_2 \leq 12$
$x_1 \geq 0, x_2 \geq 0$

ii Find optimum basic feasible solution using Modified distribution method. Use Vogel's Approximation method to obtain Initial basic feasible solution.

Factory	Customers				SUPPLY
	A	B	C	D	
P	6	4	1	1	14
Q	8	9	2	5	16
R	4	3	6	2	05
DEMAND	6	10	15	14	

B) Let $f: [a, b] \to \mathbb{R}$ be an integrable function. Let $\alpha \in \mathbb{R}$. Then prove that αf is integrable and $\int_a^b \alpha f = \alpha \int_a^b f$ **(6)**

IV. A) Answer <u>any one</u> of the following. **(6)**

i Show that the series $\sum \frac{1}{n}$ diverges.

ii Show that the series $\sum \frac{1}{n^p}$ converges whenever $p > 1$.

B) Show that the series $\sum x_n$ in $\mathbb{R}$ converges whenever $l = \lim_{n\to\infty} |\frac{x_{n+1}}{x_n}| < 1$. **(6)**

V. A) Answer <u>any one</u> of the following. **(6)**

i Show that if a sequence of real valued *continuous* functions (f_n) defined on $J \subset \mathbb{R}$ converges *uniformly* to f on J then f is also continuous.

ii Discuss the nature of convergence of the sequence of functions (f_n) given by $f_n(x) = \frac{x^n}{n}$ on [0,4]

B) State and prove Weierstrass-M test. **(6)**

VI. A) Answer <u>any one</u> of the following. **(6)**

i State and Prove Cauchy-Schwarz inequality.

ii Let $f: [a, b] \to \mathbb{R}$ be continuous and let $F(x) = \int_a^x f(t)dt$. Show that F is differentiable and $F'(x) = f(x)\ \forall x \in [a, b]$.

B) Show that an increasing bounded function f defined on $[a, b]$ is integrable. **(6)**

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