CARMEL COLLEGE OF ARTS, SCIENCE AND COMMERCE FOR WOMEN, NUVEM-GOA

SEMESTER END EXAMINATION, JUNE, 2022

Semester: II of BSC

Course Title: Matrices and Linear Algebra **Course Code: MTC102**
Total marks: 80 **Date:** **Duration: 2 hours** **Total No of pages: 2**

Instructions: *1. All questions are compulsory. However internal choice is/are available.*
2. Figures to the right indicate maximum marks to each question/sub question.
3. Use of non-programmable calculator is allowed.

Q.1. Answer <u>any four</u> of the following: **(16)**

a) Find the rank of the following matrix.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

b) Solve the following system of equations for x using Cramer's Rule.

$$2x + 3y + 4z = 9$$
$$x + y + 2z = 4$$
$$2x + z = 3$$

c) Find the adjoint of the following matrix. Also find its inverse if it exists.

$$\begin{bmatrix} 1 & 4 & 3 \\ 2 & 3 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

d) Show that the intersection of any two subspaces of a vector space V is also a subspace of V.

e) Define a line $l(p;d)$ in a vector space V, where $p \in V, d \in V\ and\ d \neq 0$.
Hence, show that $l(p;d) = l(q;d)$ if and only if $(q-p)$ is a multiple of d.

f) Show that the set $X = \{(x,y,z) \in \mathbb{R}^3 \mid 2x + 3y - z = 4\}$ is an affine space of $\mathbb{R}^3$.

Q.2. Answer <u>any four</u> of the following: **(16)**

a) Let $T: V \to V$ be a linear transformation then show that, T is one-one if and only if kernel of $T = \{0\}$

b) Let $S = \{v_1, v_2, v_3\}$ where $v_1 = (1,2,1), v_2 = (2,1,-4), v_1 = (3,-2,1)$
Verify if S is an orthogonal basis of $\mathbb{R}^3$ and express $v = (3,7,-4)$ as a linear combination of vectors v_1, v_2, v_3.

c) Suppose V is a real inner product space.
Show that $||x+y||^2 + ||x-y||^2 = 2(||x||^2 + ||y||^2)$, for any two vectors $x, y \in V$

d) Let $V = \mathbb{R}^2$
define $\langle x, y\rangle = y_1(x_1 + 2x_2) + y_2(2x_1 + 5x_2)$, where $x = (x_1, x_2)$, $y = (y_1, y_2)$
Show that this defines an inner product on $\mathbb{R}^2$.

e) Show that for a linear transformation $T: V \to V$, the eigen vectors corresponding to two distinct eigen values are linearly independent.

f) Define the following terms.
   i. Eigen Value
   ii. Eigen Vectors

Find all eigen values and their corresponding eigen vectors for the following matrix.

$$\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$$

Q.3. A. Answer <u>any one</u> of the following: **(6)**

a) Solve the following system of equations.

$$2x_1 + 3x_2 + x_3 - x_4 = 8$$
$$3x_1 + x_2 - 2x_4 = 3$$
$$x_1 + x_2 + x_3 - x_4 = 3$$
$$2x_1 + 2x_2 + x_3 + 4x_4 = 11$$

b) Reduce the following matrix to its row echelon form.

$$\begin{bmatrix} 1 & 2 & 7 & 2 \\ 2 & 3 & 1 & 3 \\ 2 & 5 & 4 & 4 \\ 1 & 3 & 4 & 3 \end{bmatrix}$$

Also comment on the nature of the solutions of the system $A\bar{x} = \bar{0}$.

B. Find the inverse of the following matrix using row reduction. **(6)**

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 2 \\ 3 & 3 & 2 \end{bmatrix}$$

Q.4. A. Answer <u>any one</u> of the following: **(6)**

a) Let S be a non-empty subset of a vector space V. Show that linear span of S is the smallest subspace of V containing S.

b) Let W be a subspace of Vector space V and let $v \in V$ be fixed. Then show that the set $S = \{v + w \mid w \in W\}$ is an affine space.

B. Let $S = \{(1,1,0,0), (0,2,2,0), (0,0,3,3)\}$ be a subset of the real vector space $\mathbb{R}^4$. **(6)**

i. Show that S is linearly independent.

ii. Write down Linear Span of S.

iii. Extend S to a basis of $\mathbb{R}^4$.

Q.5. A. Answer <u>any one</u> of the following: **(6)**

a) $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $T(-1,0,1) = (1,0,0)$, $T(0,1,-1) = (0,1,0)$ and $T(1,-1,1) = (0,0,1)$

i. Find $T(x, y, z)$

ii. Find the matrix of T with respect to the standard basis.

b) State and Prove the Rank-Nullity Theorem.

B. Define $T: \mathbb{R}^3 \to \mathbb{R}^3$ by $T(x, y, z) = (x + z, x + y + 2z, 2x + y + 3z)$ **(6)**

i. Show that T is a linear transformation

ii. Find a basis of kernel of T.

iii. Find a basis of the image of T.

Q.6. A. Answer <u>any one</u> of the following: **(6)**

a) Diagonalize the following matrix.

$$\begin{bmatrix} 1 & 3 & 4 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

b) Let V be a real inner product space.

Show that $|\langle x, y \rangle| \le \|x\|\|y\|$, for any $x, y \in V$.

B. Using the Gram-Schmidt process, transform the basis B into an orthonormal basis of $\mathbb{R}^3$ where, $B = \{(1,1,1), (-1,1,0), (1,2,1)\}$ of $\mathbb{R}^3$ **(6)**

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