Date:
Subject: Mathematics
Total marks: 80

Paper name and code: Algebra (MTC105)
Duration:2hours
Total no. of pages: 2

Instructions: i) All questions are compulsory
ii) Figures to the right indicate full marks
iii) Use of non-programmable calculators is allowed.

I) Answer <u>any four</u> of the following questions. **(4x4=16)**

a) Is the set of real numbers with respect to multiplication defines a group?

b) Let H_1 and H_2 be two subgroups of a group G. Prove that $H_1 \cap H_2$ is a subgroup of G.

c) Let G = {1, -1 , i , -i}. Is (G , .) a cyclic group?

d) Let $\alpha = (1\ 3\ 5)$ and $\beta = (2\ 4)\ (5\ 3)$ be the two permutations in S_5. Find $\alpha\beta$ and $\beta\alpha^2$.

e) Let $G = GL(2 , \mathbb{R})$. Find the Center of $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

f) Let H be a non-empty finite subset of a group G. Prove that H is a subgroup of G if and only if $a.b \in H \ \ \forall\, a, b \in H$.

II) Answer <u>any four</u> of the following questions. **(4x4=16)**

a) List the left and right cosets of H in G where $G = (\mathbb{Z}_6 , +_6)$ and $H = \{\overline{0} , \overline{3}\}$.

b) Let G = {(1), (132)(465)(78), (132)(465),(132)(456),(132)(456)(78), (78)} be the set of permutations, find stabilizer and orbit of 1, 2 and 7.

c) Let G_1 and G_2 be two groups and let $\varphi: G_1 \rightarrow G_2$ be a homomorphism. Let H be a subgroup of G_1 then prove that if H is cyclic then $\varphi(H)$ is cyclic.

d) Define Ring, Integral Domain, Ideal and Field.

e) Let $\varphi: G \rightarrow G_1$ be an isomorphism, then prove that if K_1 is a subgroup of G_1 then $\varphi^{-1}(K_1) = \{g \in G \,/\, \varphi(g) \in K_1\}$ is a subgroup of G.

f) Every integral domain is a field. Prove or disprove.

III) Answer the following questions. **(6x2=12)**

A) Answer <u>any one</u> of the following.

i) Let (G, .) be a group and let H be a non-empty subset of G then prove that H is a subgroup of G if and only if $a.b^{-1} \in H \ \ \forall\, a, b \in H$.

ii) Prove that any subgroup of a cyclic group is cyclic.

B) Prove that $o(HK) = o(H).\ o(K)/o(H \cap K)$

IV) Answer the following questions. **(6x2=12)**

A) Answer <u>any one</u> of the following.

i) Prove that the order of a permutation of a finite set written in disjoint cycle form is the least common multiple of the length of the cycles.

ii) Prove that every permutation of a finite set can be written as cycle or as a product of disjoint cycle.

B) Prove that every group is isomorphic to a group of permutations. (Cayley's

V) Answer the following questions. **(6x2=12)**

A) Answer <u>any one</u> of the following.

i) For $n > 1$, prove that An has order $n!/2$.

ii) Let G be a finite abelian group and let p be a prime that divides the order of G. Prove that G has an element of order p.

B) State and prove Langrange's theorem.

VI) Answer the following questions. **(6x2=12)**

A) Answer <u>any one</u> of the following.

i) Let $\varphi: R \rightarrow S$ be a ring homorphism, Let A be an ideal of R and φ is onto, then prove that $\varphi(A)$ is an ideal of S.

ii) Prove that intersection of an arbitrary family of ideals of a ring R is an ideal of R.

B) Prove that a non-empty subset S of a ring $(R, +, .)$ is a subring iff $a - b \in S$ and $a.b \in S \quad \forall\, a, b \in S$.

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