Subject: Mathematics

Paper name and code: Calculus of 2 and 3 variables (MTC 107)

Duration: 2hr **Total marks: 80** **Total number of pages: 02**

Instructions: *i) All questions are compulsory*

ii) Figures to the right indicate full marks.

iii) Use of non- programmable calculator is allowed.

Q1. Answer ANY FOUR of the following: (4x4mks=16)

a. Compute the directional derivative of $(x, y, z) = x + yz + z^2$ at a point (1, -2, 1) in the direction of (-1, 1, 0).

b. State and verify Cauchy Schwarz inequality and triangle inequality for $\bar{x}$=(1,3,0,1) $and\ \bar{y} = (-1,0,1,1)$.

c. Show that $f(x, y) = \frac{x+y}{x-y}$ is continuous at (1,2).

d. Determine the velocity and acceleration vector to the curve $(t) = (4e^t, 6t^3, \sin t)$ $at\ t = 2$.

e. For any C^2 function f, Prove that $Curl(grad\ f) = 0$

f. Find the Jacobian matrix for $f(x, y) = (e^{x+y}, y + e^x)$ at (0, 1).

Q2. Answer ANY FOUR of the following: (4 x 4mks=16)

a. Determine whether or not the vector field F = (y + z) i + (x + z) j + (x + y) k is irrotational, and if so, find a scalar potential function for it.

b. Use triple integral to find the volume of the box [-3,1] X [1 ,2] X [0 ,3]

c. Evaluate the line integral $\int xy\,dx + x^2y\,dy + z\,dz$ along the path $(t) = (t, t^2, 1)$ from t=0 to t=2.

d. Evaluate $\int_0^1 \int_0^x 2x + y^2\ dydx$.

e. Use Gauss divergence theorem to evaluate $\iint_S F.dS$, where S is the surface defined by $x^2 + y^2 \leq 1, -1 \leq z \leq 1$

f. Find the work done by the force field F(x,y)=$(x^2 + y^2)(i + j)$ when a particle is moved along a straight line joining (0,0,1) to (3, 1, 1).

Q3.A. Answer ANY ONE of the following: (6)

i. Show that the function $f(x, y) = \frac{xy}{\sqrt{x^2+y^2}}$ is differentiable. Determine whether it is C'.

ii. Determine the second order Taylor's formula for $f(x, y) = e^{x+y}$ about (0,0)

B. Let $\bar{r} = (x, y, z)$ $and\ r = \sqrt{x^2 + y^2 + z^2}$. Prove that $\nabla^2 r^n = (n+1)r^{n-2}$ (6)

Q4. A. Answer <u>ANY ONE</u> of the following: (6)

i. Find the maximum and the minimum value of $(x, y) = x^2 + y^2 + 6x - 4y + 13$.

ii. Maximize the function $f(x, y, z) = x + z$ subject to the constraint $x^2 + y^2 + z^2 = 1$

B. Find f_x, f_y at (0,0) where $f(x,y) = \begin{cases} \frac{x^2-y^2}{x+y}; (x,y) \neq (0,0) \\ 0 \quad ; (x,y) = (0,0) \end{cases}$. Check whether f is continuous at (0,0) (6)

Q5.A. Answer <u>ANY ONE</u> of the following: (6)

i. Evaluate $\iint_D x + y\, dxdy$ where D is the region bounded by y=x and $y=x^2$

ii. Evaluate using cylindrical coordinates $\iiint x + y\, dxdydz$ over the volume enclosed by $x^2 + y^2 = 1$, $z = 3$ *and* $z = 0$

B. Find the average value of $f(x,y)= x+y^2$, over the triangle bounded by (0,0), (1,1) and (2,0).

(6)

Q6.A. Answer <u>ANY ONE</u> of the following: (6)

i. Evaluate $\iint_D x^2 - y^2\, dxdy$ where D is a square with vertices (0,0), (1,-1), (1,1) and (2,0) by changing the variables as x=u+v and y=u-v.

ii. Use Stokes's Theorem to find the value of $\iint f.\, dS$ where, $f(x, y, z) = xy\hat{i} + yz\hat{j} + xz\hat{k}$ on the surface x+y+z= 4

B. Verify Green's theorem for $(x, y) = xy\,\hat{i} + (y + x)\,\hat{j}$ in the region bounded by $y = x$, $x=2$ *and* $y = 0$ (6)