CARMEL COLLEGE OF ARTS, SCIENCE & COMMERCE FOR WOMEN, NUVEM-GOA

SEMESTER END EXAMINATION, JANUARY 2022

Semester: V **Quantum Mechanics** **PYD 101**

Total Marks:80 **Date:**22/01/2022 **Duration:**2 hours **Total Number of Pages:**02

Instructions:

1) *All questions are compulsory, however internal choice is available.*
2) *Figures to the right indicate maximum marks to the question.*
3) *Symbols have their usual meanings unless otherwise stated.*
4) *Draw neat diagram wherever necessary.*
5) *Use of non-programmable calculator is permitted.*

Constants: $h = 6.63 \times 10^{-34}\, J.sec.$ $e = 1.6 \times 10^{-19}\, C.$
$m_e = 9.1 \times 10^{-31} kg.$ $m_p = 1.66 \times 10^{-27}\, kg.$

Q1. Answer **any FOUR** of the following questions. *(4 x 4 = 16)*

A) Obtain Bohr's postulate regarding quantization of angular momentum of the electron in hydrogen atom on the basis of stationary de Broglie waves.

B) Discuss the Davisson and Germer experiment and explain it's importance.

C) Write the requirements for a wave function ψ, to be considered acceptable.

D) Consider a non-relativistic free particle having velocity v, what is it's phase velocity and group velocity?

E) Illustrate Heisenberg's Gamma Ray Microscope experiment.

F) Show that the dispersion relation is given by $v_g = v_p - \lambda \frac{dv_p}{d\lambda}$.

Q2. Answer **any FOUR** of the following questions. *(4 x 4 = 16)*

A) Discuss the basic postulates of quantum mechanics.

B) Show that for a particle moving in a circle at any instant, $\Delta L.\Delta\theta \geq h/2\pi$ where ΔL *and* $\Delta\theta$ are the uncertainties in angular momentum and angular position.

C) Find the expectation value of position of a particle if wavefunction $\psi(x) = ax$ between x=0 and x=1 and zero everywhere else.

D) A particle is described by the wavefunction $\psi = Ax(a-x)$ *such that* $0 < x < a$. Find A?

E) An α particle of energy 40 eV hits a potential barrier of 30 eV. Find the reflective and transmission coefficient.

F) Write a short note on α decay using the theory of tunnel effect.

Q3. A) p) State and explain de Broglie hypothesis. *3*

q) What do you mean by normalization of a wave function. *3*

OR

x) Explain the terms group velocity and phase velocity. *3*

y) Find the de Broglie wavelength of an electron having kinetic energy 1 eV. *3*

B) Show that the velocity (v) of a particle of mass m is same as group velocity (v_g). Also comment about the velocity of de Broglie waves. *6*

Q4. A) p) State Heisenberg's Uncertainty principle. *3*

q) Compare uncertainties in the velocities of an electron and a proton confined in a 10 Å box. *3*

OR

x) Discuss Max Born's interpretation of wave function. *3*

y) Calculate the uncertainty in the position of an electron if the uncertainty in it's momentum is $1.65 \times 10^{-24} kg.m/s$. *3*

B) Derive the general one-dimensional time dependent Schrödinger's equation. *6*

Q5. A) p) Consider the wavefunction of a particle moving in one dimension is given $\psi = Ae^{-x^4}$ with energy eigen value E=0 and $\hbar = 2m = 1$. Find the potential energy of the system. *3*

q) Find $[ln\ x, \hat{p}_x]$ *3*

OR

x) The eigenfunction of the operator $\frac{d^2}{dx^2}$ is sin 2x. Find the corresponding eigen value. *3*

y) For the wavefunction $\psi = Ae^{ikx} + Be^{-ikx}$. Find the flux (J). *3*

B) Determine the transmission and Reflective coefficient for a particle moving in a one-dimensional potential given by V=0 for x<0 and V=V_0 for x>0, for the energy of the particle E<V_0. *6*

Q6. A) p) What do you mean by observable and operators in quantum mechanics? Give some examples of quantum mechanical operators. *3*

q) For a particle in a 1D box of length L, find the probability of finding the particle between 0 <x <L/2 of the box. 3

OR

x) Verify the operator equation $\left(\frac{d}{dx} - \alpha x\right)\left(\frac{d}{dx} + \alpha x\right) = \frac{d^2}{dx^2} - \alpha^2 x^2 + \alpha.$ 3

Given that α is a constant.

y) Write the wavefunction and energy and for a particle in a cubical box. Does this cubical box exhibit degeneracy? If yes, how can it be removed? 3

B) Solve Schrödinger's wave equation for a particle of mass m in one dimensional potential well of width L. Write the normalized eigen functions and eigen values of the lowest state. 6
