**CARMEL COLLEGE OF ARTS SCIENCE AND COMMERCE FOR WOMEN, NUVEM-GOA**

**B.Sc CBCS Semester V (Regular)**

**January 2022**

**Subject Code: MTE 102** **Subject Name: Combinatorics**

**Total marks: 80** **Duration: 2hrs** **Total no. of Pages: 3**

***Instructions:*** *i) All questions are compulsory.*

*ii) Figures to the right indicate full marks.*

*iii) Use of non-programmable calculators is allowed.*

*Notations:* *[n] – The set of first n positive integers.*

*S(n,k) – Number of partitions of the set [n] into k parts.*

*c(n,k) – Number of permutations of the set [n] having k cycles.*

**I. Answer <u>any four</u> of the following.** **(4x4 = 16)**

1. Ten points are given within a unit square. Show that there are 2 points closer to each other than 0.48 units and 3 points that can be covered by a disc of radius 0.5 units.
2. If 11 positive integers less than 29 are selected at random, prove that there will be at least two among them that have a common divisor larger than 1.
3. Show that the number of subsets of $[n]$ is $2^n$.
4. For all positive integers $n$, show that
$$\sum_{k=0}^{n} \binom{n}{k} = 2^n$$
5. Show that the number of partitions of the positive integer $n$ into $k$ parts is equal to the number of partitions of $n$ whose largest part is of size $k$.
6. Show that $S(n, 2) = 2^{n-1} - 1$.

**II. Answer <u>any four</u> of the following.** **(4x4 = 16)**

1. Let n be a positive integer. Use the principle of mathematical induction to prove that it is possible to cut up a cube into 7n+1 smaller cubes.
2. How many permutations of 15 have three 2-cycles, four 1-cycles and one 5 cycle?
3. Obtain $c(4,2)$.
4. How many positive integers $k \leq 210$ are relatively prime to 210?
5. How many functions are there from $[n]$ to $[n]$ that are not one-one?
6. Let $\{a_n\}_{n\geq 0}$ be a sequence such that $a_0 = 0$ and $a_n = a_{n-1} + 4$. Obtain a closed formula for $a_n$.

interest at the end of each year. At the beginning of each year, we deposit another Rs. 500 into this account. How much money will be there in this account after n-years? **(6)**

**VI.** **A) Answer <u>any one</u> of the following.** **(6)**

i. Let $a_n$ be the number of ways to build a certain structure on an n-element set, and let $b_n$ be the number of ways to build another structure on an n-element set. Let $c_n$ be the number of ways to separate $[n]$ into the intervals $S = \{1,2,\dots,i\}$ and $T = \{i+1, i+2, \dots, n\}$ (S and T allowed to be empty), and then to build a structure of the first kind on $S$ and a structure of the second kind on $T$. Let $A(x), B(x)$ and $C(x)$ be the respective generating functions of the sequences $\{a_n\}, \{b_n\}, \{c_n\}$. Then show that

$$A(x)B(x) = C(x)$$

ii. Let $a_n$ be the number of ways to build a certain structure on an n-element set, and let $b_n$ be the number of ways to build another structure on an n-element set. Let $c_n$ be the number of ways to separate $[n]$ into the disjoint subsets $S$ and $T$, $(S \cup T = [n])$, and then to build a structure of the first kind on $S$, and a structure of the second kind on $T$. If A(x), B(x) and C(x) are the respective generating functions of the sequences $\{a_n\}, \{b_n\}, \{c_n\}$, then show that $A(x)B(x) = C(x)$.

**B)** For all positive integers $m$, show that $|ODD(2m)| = |EVEN(2m)|$ **(6)**

******************************