CARMEL COLLEGE OF ARTS, SCIENCE AND COMMERCE FOR WOMEN, NUVEM-GOA

B.Sc. CBCS Semester V (Regular)Examination, January/February, 2022

Subject Code: (MTE101) **Subject Name: Foundations of Mathematics**

Total marks: 80 **Duration: 2 Hrs** **Total No. of pages: 2**

Instructions: *1. All questions are compulsory, however internal choice is available.*
2. Figures to the right indicate maximum marks allotted to the question.
3. Use of Scientific non-programmable calculator is allowed.

Q.1. Attempt **any four** of the following: **[16]**

a) "There exists an integer $x \in \mathbb{Z}$ such that for any $y \in \mathbb{Z}$ we have $x + y = y$". Write this in symbols using quantifiers, negate it and write the negation in words.

b) Write the converse and the contrapositive of the statement. "For real numbers x and y, if xy is an irrational number then x is irrational or y is irrational.

c) Identify the set $A = \{x \in \mathbb{R} : \big||x-2| - |x-4|\big| = 1\}$.

d) Let $f: X \to Y$ and $g: Y \to Z$ be such that the composition $(g \circ f)$ is injective and f is surjective. Show that g is injective.

e) Prove the following statement,
"For an integer n, if $n^3 - 1$ is even, then n is odd."

f) For any three sets A, B, C, Show that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

Q.2. Attempt **any four** of the following: **[16]**

a) Using Induction Principles, prove that $n! > 2^n$ for all positive integers $n \geq 4$.

b) For $n \in \mathbb{N}$, define a_n as follows: $a_1 = 1, a_2 = 2, a_3 = 3$ and for $n \geq 3$
$a_n = a_{n-1} + a_{n-2} + a_{n-3}$. Show that $a_n < 2^n$ for each $n \in \mathbb{N}$.

c) Show that "A subset of a countable set is countable".

d) Prove the statement using the Induction Principles "Every integer greater than 1 has a prime divisor".

e) We define a relation $\preccurlyeq$ on   as, $a \preccurlyeq b$ if and only if $a \leq 2b$.
Is this relation reflexive, symmetric, anti-symmetric, transitive?

f) A be a subset of a partially ordered set $(X, \leq)$, define the following terms:

i. Chain
ii. Lower bound of A.
iii. Maximum of A
iv. Infimum of A

Q.3.A. Attempt **any one** of the following: **[6]**

a. Prove the statement, "For $a, b \in \mathbb{Z},\ a^2 - 4b \neq 2$."

b. Find whether the function $f: [0,1] \to [0,1], f(x) = (1-x)/(1+x)$ is bijection.

B. Define the complement of a set.
Hence, State and prove the De Morgan's Theorem. **[6]**

Q.4.A. Attempt **any one** of the following: **[6]**

a. Let $f: X \to Y$, $g: Y \to Z$ and $h: Z \to W$ be functions, show that

$$h \circ (g \circ f) = (h \circ g) \circ f$$

b. Let $f: X \to Y$ and $\mathcal{F} = \{A_\alpha \subseteq Y: \alpha \in I\}$ a family of subsets of Y indexed by I. Then show that $f^{-1}(\bigcup_{\alpha \in I} A_\alpha) = \bigcup_{\alpha \in I} f^{-1}(A_\alpha)$.

B. If $f: X \to Y$ and $g: Y \to Z$ are bijection, then $g \circ f$ is a bijection and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ **[6]**

Q.5.A. Attempt **any one** of the following: **[6]**

a. State the Induction Principle and the Strong Induction Principle.
Hence show that Induction Principle implies the Strong Induction Principle.

b. A relation $\sim$ on $\mathbb{Z} \times \mathbb{Z}^*$, where $\mathbb{Z}^* = \mathbb{Z} \setminus \{0\}$ is given by $(a, b) \sim (c, d)$ if and only if $ad = bc$. Show that $\sim$ is an equivalence relation. Hence, find the equivalence classes and a suitable transversal.

B. State and prove the Schröeder-Berstain Theorem. **[6]**

Q.6.A. Attempt **any one** of the following: **[6]**

a. Prove that if A is finite and B is a proper subset of A, then $|B| < |A|$

b. For a non-empty set A, show that the following are equivalent.

i. A is countable.
ii. There is a one-one map of A into $\mathbb{N}$.
iii. There is an onto map from $\mathbb{N}$ onto A.

B. Let $(X, \leq)$ and $(Y, \leq)$ be partially ordered sets. On $X \times Y$,define a relation $(x_1, y_1) \preccurlyeq (x_2, y_2)$ if and only if either $x_1 < x_2$ or ($x_1 = x_2$ and $y_1 \leq y_2$)
Show that this relation on $X \times Y$ is a partial order. **[6]**
