PAPER NAME AND PAPER CODE: **ANALYSIS II (MTC 106)** No. OF PAGES: **02**

Instructions:

1. **All questions are compulsory, however internal choice is available.**
2. **Figures to the right indicate the marks allotted to the questions.**
3. **Use of Non-programmable calculator is allowed.**

Q1. Answer any Four of the following **(4*4 = 16)**

1. Prove that $\beta(m,n)=\beta(n,m)$ where β denotes beta function.
2. Prove that $\Gamma(n+1)=n\,\Gamma n$ where Γ denotes gamma function .
3. Find the radius of convergence and interval of convergence of the power series $\sum_{n=1}^{\infty}\frac{x^n}{n!}$.
4. Discuss the convergence of $\int_0^{\infty} e^{-x}\frac{\sin x}{x}dx$.
5. Discuss the convergence of $\int_0^1 \frac{\sec x}{x}dx$.
6. Give an example of the power series with R = 5 that diverges at x = - 5 and converges at x = 5.

Q2. Answer any Four of the following **(4*4 = 16)**

1. If $f(t) = 3t^2 - 4,\ g(t) = t^2$ in the polynomial space P(t) with usual integral inner product $< f,g >= \int_0^1 f(t)g(t)dt$
   Find (i) $< f,g >$
   (ii) $\|f(t)\|$
2. Find the constants A and B if the set $S = \{1,\ x,\ 2 + 3Ax + Bx^2\}$ is orthogonal in C [-1,1] with usual integral inner product.
3. Let f and g be functions in C [0,1] under usual integral inner product, Show that $\|f\|^2\|g\|^2 - |< f,g >|^2 \geq 0$
4. Find the Fourier series of f(x)=x on [-π, π]
5. Obtain the expression for a Fourier series of an odd function.
6. Let (V, <,>) be an inner product space over a field F and $\|x\| = \sqrt{< x,x >}$ for all x in V, be a norm induced by the inner product on V. Prove that $\|x + y\|^2 + \|x - y\|^2 = 2(\|x\| + \|y\|)$ for all x, y in V.

Q3. Answer the following

A. Answer any one of the following **(6 marks)**

1 State and prove Parseval's theorem.

Q3. Answer the following

(6 marks)

A. Answer <u>any one</u> of the following

1 State and prove Parseval's theorem.

2. Let $\varnothing = \{\varnothing_1, \varnothing_2, \varnothing_3, \ldots\}$ be an orthonormal set in C[a,b] with respect to the usual integral inner product on C[a,b].

If $f \in C[a,b]$ and if $c_k = \langle f, \emptyset_k \rangle$ then prove that $\sum_{k=1}^{\infty} c_k^2 \leq \|f\|^2$.

B. Prove that $\{sinx, cosx, 1\}$ is an orthogonal set of functions on $C[-\pi, \pi]$ with the usual integral inner product. And hence determine the corresponding orthonormal set. **(6 marks)**

Q4. Answer the following

A. Answer <u>any one</u> of the following **(6 marks)**

1. Find the Fourier series of $f(x) = \begin{cases} x - \pi & -\pi \leq x \leq \pi \\ -x - \pi & 0 \leq x \leq \pi \end{cases}$ on $[-\pi, \pi]$

2. Find the Fourier series of $f(x)=x^2$, $x \in [-\pi, \pi]$ and deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$

B. Find the Fourier cosine series of $f(x)= x - x^2; x \in [0,1]$. **(6 marks)**

Q5. Answer the following

A. Answer <u>any one</u> of the following. **(6 marks)**

1. Prove that $\int_0^\infty \frac{dx}{x^p}$ is convergent iff $p > 1$.

2. Let $a, x_0 \in \mathbb{R}$ such that $x_0 > 0$. Let a real number k>o be such that $|f(x)| \leq k|g(x)| \;\; \forall\, x \geq x_0$. If $\int_a^\infty |g(x)|\,dx$ is convergent then prove that $\int_a^\infty |f(x)|\,dx$ is convergent.

B. Prove that $1 \leq \frac{9}{\pi^2} \int_{\pi/6}^{\pi/2} \frac{x}{\sin x}\,dx \leq 2$.

(6 marks)

Q6. Answer the following

A. Answer <u>any one</u> of the following. **(6 marks)**

1. Prove that $s'(x) = c(x)$ where $s(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ and $c(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$.

2. Prove that the improper integral $\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$ converges if and only if $m > 0$ and $n > 0$.

B. For any positive integer m prove that **(6 marks)**

$2^{2m-1}\Gamma(m)\Gamma(m+\frac{1}{2}) = \Gamma(\pi)\Gamma(2m)$ where Γ represents gamma function.

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