CARMEL COLLEGE OF ARTS, SCIENCE & COMMERCE,

NUVEM – GOA.

B.Sc. CBCS Semester V (Regular) Examination, January 2022

Sub. Code:PYC107 Sub: Mathematical Physics & ElectromagneticTheory I

Total marks: 80 Duration: 2 hrs Total no. of pages: 03

Instructions:

1) *All questions are compulsory. Internal* choices are *available.*
2) *Figures to the right indicate full marks*
3) *Symbols have their usual meaning, unless otherwise stated.*

Q 1. Answer any four of the following: **(4x4=16)**

a) Prove that the divergence of curl of a vector is equal to zero.

b) A fluid motion is given by $\vec{v} = (ysinz - sinx)i + (xsinx - 2yz)j + (xycosz + y^2)k$, check out if the motion is irrotational or not.

c) A vector field is given by $\vec{F} = (2y + 3)\hat{\imath} + xzj + (yz - x)\hat{k}$. Evaluate $\int \vec{F}.\overrightarrow{dr}$ along the path c where $x = 2t, y = t, z = t^3$ from $t = 0$ to $t = 1$.

d) Prove that $\vec{\nabla}.(\vec{F}x\vec{G} = \overrightarrow{G}.(\vec{\nabla}x\vec{F}) - \overrightarrow{F}.(\vec{\nabla}x\vec{G})$

e) Show that force between two charges separated by a distance is reduced by a factor $\left[\frac{1}{1+\frac{P}{(\epsilon_0 E)}}\right]$ due to presence of a dielectric between them.

f) The dielectric constant of a medium is 4. Electric field in the dielectric is 10^6 V/m. Calculate electric displacement and polarisation.

Q 2. Answer any four of the following: **(4x4=16)**

a) Solve the differential equation $(x + y - 10)dx + (x - y - 2)dy = 0.$

b) Use the chain rule to solve: 1) $\frac{d}{dx}\{\ln(sinx^2\}$ 2) $\frac{d}{dx}\{(cos\sqrt{2x^2+2}\}$

c) Two semi- infinite planes at φ=0 and $\varphi = \frac{\pi}{6}$ are separated by a gap. If V(φ=0) =0 and V($\varphi = \frac{\pi}{6}$) =100V. Calculate V and E in the region between the plates.

d) Calculate the work done to assemble charges of 0.5μC,10 μC and -70 μC at the corners of an equilateral triangle.

e) Capacitance of a parallel plate capacitor is 400picofarad and its plates are separated by distance of 2mm of air. What will be its energy when it is charged to 1500 volts? What will be its potential difference with the same charge if plate separation is doubled?

f) An electric line of force is going from one dielectric (K=5) to another dielectric(K=4). If it makes an angle of 60° with the boundary in the first dielectric, what is its direction in the second dielectric.

Q.3

A) Solve: $\{2xycosx^2 - 2xy + 1\}dx + \{sinx^2 - x^2 + 3\}dy = 0$ **6**

OR

A) Apply the method of separation of variables to find the solution to the partial differential equation $3u_x + 2u_y = 0,$

given $u_x = \frac{\partial u}{\partial x}$ and $u_y = \frac{\partial u}{\partial y}.$

B) Check if the following is an exact differential, and hence obtain its solution

$(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3 - 3x^2y^2 - 5y^4)dy = 0$

6

Q.4

A) Write Laplace's equation for one dimension in cylindrical coordinates and obtain its solution. **6**

OR

A) a) State any two properties of Dirac delta functions.

b) Evaluate $\int_o^\infty e^{-3t}\,\delta(t-5)dt$ and $\int_{-\infty}^{\infty} cos2t\delta\left(t-\frac{\pi}{4}\right)dt$

B) In a cylindrical capacitor, V=0 if ρ=a
V=V_0 if ρ=b(outer radius)
obtain expression for its capacitance using solutions to Laplace equation

6

Q.5

A) A point charge 2q is kept at a distance d from a grounded conducting plane, and another charge q is also placed at a distance 2d from the same conducting plane on the same side use the method of images to obtain the force exerted on charge q.

6

OR

A) Explain the term electric susceptibility. A point charge q is situated in a dielectric medium of dielectric constant k. Derive expression relating electric displacement and polarisation.

B) Derive an expression for electric potential due to a dipole at any arbitrary point

6

Q.6

A) Derive integral form of Gauss' law in dielectric in terms of electric displacement.

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OR

A) Derive the Langevin -Debye equation in the study of polar dielectrics.

B) Deduce the boundary conditions to be satisfied by the electric field vector at the interface of two dielectric media.

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