CARMEL COLLEGE OF ARTS SCIENCE AND COMMERCE FOR WOMEN
NUVEM-GOA
SEMESTER END EXAMINATION JANUARY 2022

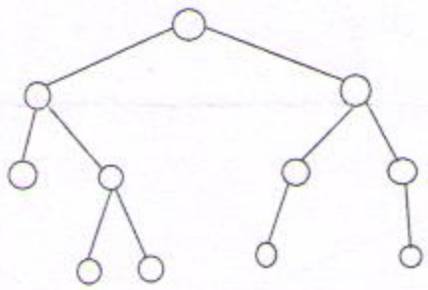
Semester: III **Course name and code: Differential Equations and Discrete Mathematics (MTC103)**

Total marks: 80 **Date:** **Duration: 2 hrs** **Total number of pages: 3**

Instructions: *i) All questions are compulsory.*
ii) Figures to the right indicate full marks.
iii) Use of non programmable calculators is allowed.

I) Answer any four of the following questions. **(4x4=16)**

1. Find a general solution for the following differential equation.
$$(x^2 - 2y^2)dx + xy\,dy = 0$$
2. Check whether the following first order differential equation is exact. If yes then find its general solution.
$$(y - x^3)dx + (x + y^3)dy = 0$$
3. Solve the following differential equation
$$y^2p^3 - 2xp + y = 0$$
where $p = \frac{dy}{dx}$
4. Find the general solution for the following differential equation.
$$(x^3 + xy^3)dx + 3y^2dy = 0$$
5. Consider a connected graph G. Prove that the cut set and any spanning tree of G must have at least one edge in common.
6. Define the terms
   i. Binary Tree
   ii. Path length
   Obtain a binary prefix code for the binary tree below.



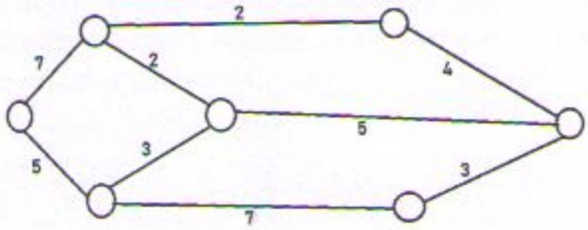
II) Answer any four of the following questions. **(4x4=16)**

1. Solve the following differential equation
$$\frac{dy}{dx} - \frac{y}{x} = y^9$$
2. Show that any linear combination of two solutions of a second order homogeneous linear differential equation is also a solution.
3. Show that e^x and xe^x are two linearly independent solutions of
$$y'' - 2y' + y = 0$$

4. Find the general solution of $y''' + y = 0$.

5. Use the D-operator method to find a solution of $\frac{dy}{dx} - y = x^3$.

6. Find the minimum spanning tree in the following weighted graph.



III) A) Answer any one of the following. **(6)**

i) Solve the following differential equation

$$(y - xy^2)dx + (x + x^2y^2)dy = 0$$

ii) An electric circuit has in series an EMF given by $E = 100\sin(40t)$ $Volts$, a resistor of 10Ω and an inductor of 0.5 H. If the initial current is 0, find the current at time t

B) Given that y_1 and y_2 are two solutions of the differential equation $y'' + py' + qy = 0$, obtain a solution to the non-homogeneous differential equation $y'' + py' + qy = R(x)$. **(6)**

IV) A) Answer any one of the following. **(6)**

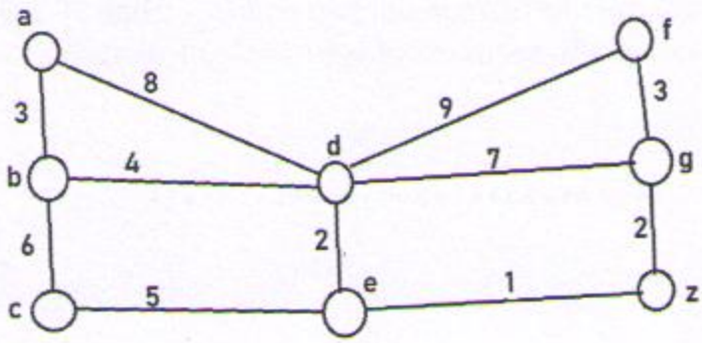
i. Solve the following differential equation using the method of undetermined coefficients.

$$y'' + 4y = 4\cos(2x) + 6\cos x + 8x^2 - 8x$$

ii. Solve the following differential equation using the method of variation of parameters where $y(0) = 0$ and $y'(0) = 1$

$$y'' + 2y' + y = e^{-x}\log(x)$$

B) Use Dijkstra's Algorithm to find the shortest path between a and z. **(6)**



V) **A) Answer any one of the following.** **(6)**

i. If y_1 and y_2 are two solutions of $y'' + P(x)y' + Q(x)y = 0$ on $[a, b]$, then show that they are linearly dependent on this interval if and only if their Wronskian is identically equal to 0.

ii. State the existence and uniqueness theorem of solutions of a system of linear first order differential equations. Find the solution for the following system of differential equations.

$$\frac{dx}{dt} = 5x + 4y,$$
$$\frac{dy}{dt} = -x + y$$

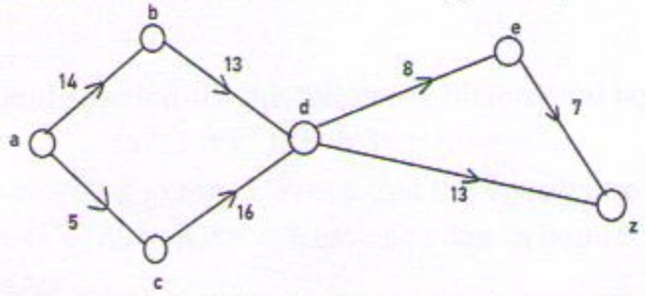
Where $y(0) = 0$ and $x(0) = 1$

B) Find a general solution of the differential equation. **(6)**

$$y''' + 2y'' - y' = 3x^2 - 2x + 1 + 5x^2e^{4x}$$

VI) **A) Answer any one of the following.** **(6)**

i. Find the maximum flow in the following transport network



ii. Find the general solution for the following system of equations.

$$\frac{dx}{dt} = x + 2y + 2e^{4t}$$
$$\frac{dy}{dt} = 2x + y + e^{4t}$$

B) Let T be a tree with 60 edges. The removal of a certain edge from T yields 2 disjoint trees T_1 and T_2. Given that the number of vertices in T_1 is equal to the number of edges in T_2, determine the number of vertices in T_1 and T_2 **(6)**

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