CARMEL COLLEGE OF ARTS, SCIENCE AND COMMERCE FOR WOMEN, NUVEM-GOA

SEMESTER END EXAMINATION, JANUARY, 2022

Semester: I OF B.Sc Course name & code: CALCULUS AND NUMERICAL METHODS
Total marks: 80 Date: Duration: 2 hours Total No of pages: 2

Instructions: *1. All questions are compulsory.*
2. Figures to the right indicate maximum marks to the questions.
3. Use of non-programmable calculators is allowed.

Q.1. Answer any FOUR of the following: **($4\times 4 = 16$ marks)**

a) For $x,\ y \in \mathbb{R}$, define '$x\ is\ less\ than\ y$'
If $x < y$ and $z > 0$ then show that $xz < yz$

b) For $x,\ y \in \mathbb{R}$, show that $|x| \leq y$ if and only if $-y \leq x \leq y$

c) Prove that, "Every convergent sequence is bounded"

d) Show that, the sequence $\langle x_n \rangle$ defined as $x_n = \frac{n^2-1}{n^2+1}$ converges to 1, state the results used.

e) Prove that: If x_n converges to c then $|x_n|$ converges to $|c|$

f) Find the nth derivative of $f(x) = e^{ax} \cos\ (bx + c)$

Q.2. Answer any FOUR of the following: **($4\times 4 = 16$ marks)**

a) Evaluate $\lim\limits_{x\to 0} \frac{x^2-\sin^2 x}{x^4}$

b) Discuss the continuity and differentiability of the function $f\colon \mathbb{R} \to \mathbb{R}$ given by $f(x) = |2x - 1|$ at $x = {}^1/_2$

c) Verify the Lagrange's Mean value theorem for the following function $f(x) = 2x^2 - 7x + 10$ on $[2,5]$

d) Show that the function $f\colon \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^3 - 9x^2 + 30x + 13$ is increasing in $\mathbb{R}$.

e) Explain the method to obtain the root of an algebraic equation using Bisection Method and also give its geometrical interpretation.

f) Find the root of $x^2 - 5x + 2 = 0$ correct to 2 decimal places by Newton-Raphson Method.

Q.3. A. Answer any ONE of the following: **($6\times 2 = 12$ marks)**

i. If A and B are bounded subsets of $\mathbb{R}$, Show that $A \cup B$ and $A \cap B$ are also bounded.

ii. For real number x define the absolute value, hence
Show that for all real numbers x and y, $|x+y| \le |x| + |y|$

B. State and prove the Fixed-Point Theorem for real valued continuous functions.

Q.4. A. Answer any ONE of the following: **($6\times 2 = 12$ marks)**

i. If a sequence converges, it converges to a unique limit.

ii. If $\langle x_n \rangle$ and $\langle y_n \rangle$ are two sequences such that $x_n \to x$ and $y_n \to y$, then $x_n + y_n \to x + y$.

B. Show that a bounded monotonic increasing sequence of real numbers is convergent.

Q.5. A. Answer any ONE of the following: **($6\times 2 = 12$ marks)**

i. A curve is drawn to pass through the points given by the following table

x:	1	1.5	2	2.5	3	3.5	4
y:	2	2.4	2.7	2.8	3	2.6	2.1

Estimate the area bounded by the curve, the X-axis and the lines x=1, x=4 using Simpson's $1/3^{rd}$ rule.

ii. State and Derive Trapezoidal Rule.

B. Prove that, If a function f is continuous on a closed interval $[a, b]$, then it *attains its bounds* at least once in $[a, b]$

Q.6. A. Answer any ONE of the following: **($6\times 2 = 12$ marks)**

i. Use Taylor's theorem to show that $e^x \cos x = 1 + x - \frac{2x^3}{3!} - \frac{2^2x^4}{4!} - \frac{2^2x^5}{5!} + \cdots$

ii. Find the maximum and minimum values of $f(x) = 2x^3 - 15x^2 + 36x + 10$.

B. State and prove Rolle's Theorem.