

Carmel College Arts Science and Commerce for Women, Nuvem Goa

B.Sc. CBCS Semester IV Regular Examination July 2021

Subject: Mathematics

Paper name and code: Analysis and Operations research (DSC 1D)

Duration: 2hr

Total marks: 40

total number of pages: 03

Instructions: i) All questions are compulsory
ii) Figures to the right indicate full marks.

Q1. Answer ANY FIVE of the following:

(5 X 2 mks)

a. When is a series said to be absolutely convergent? Does $\sum_{n=1}^{\infty} \frac{(-1)^{n+2}}{n^2}$ converge absolutely.

b. Does $\sum_{n=2}^{\infty} \frac{1}{n^3 \log n}$ converge?

c. State True or False. Justify your answer.

Every convergent series is absolutely convergent.

d. Show that if f is integrable on $[a, b]$ then f^2 is integrable on $[a, b]$.

e. State True or False. Justify your answer.

Every bounded function on $[a, b]$ is Riemann integrable on $[a, b]$.

f. Find the pointwise limit of the $\{f_n\}$ defined by $f_n(x) = \frac{x^n}{1+x^n}$ $x \in [0, 2]$

g. A certain item has a uniform demand rate of 24,000 units per year. The demand is fixed and shortages are not allowed. The inventory holding cost is Rs.0.40 per unit per month and the set-up cost per run is Rs. 300/- per run.

Determine (a) the optimal lot size, (b) Optimum time between production runs.

- h. Find the dual of: Minimize , $Z = x_1 - 3x_2 - 2x_3$
 Subject to $3x_1 - x_2 + 2x_3 \leq 7$;
 $2x_1 - 4x_2 = 12$;
 $3x_1 + 3x_2 + 8x_3 \geq 10$;
 $x_1 \geq 0, x_2 \geq 0, x_3$ unrestricted

Q2. Answer ANY SIX of the following:

(4 X 5 mks)

- a. Test for convergence the following series:

i) $\sum_{n=1}^{\infty} e^{-n^2}$

ii) $\sum_{n=1}^{\infty} \frac{3^{1-2n}}{n^2+1}$

- b. Let f and g be differentiable functions with f' and g' as Riemann integrable on $[a, b]$ then prove that

$$\int_a^b f(x)g'(x)dx + \int_a^b f'(x)g(x)dx = f(b)g(b) - f(a)g(a).$$

- c. Determine two partitions P and Q of the interval $[0,1]$ such that Q is the refinement of P . Given $f: [0,1] \rightarrow R, f(x) = x^3$, find the upper Riemann sum and the lower Riemann sum corresponding to each of the partitions P and Q . Hence show that $L(P, f) \leq L(Q, f)$ and $U(Q, f) \leq U(P, f)$.

- d. Show that $\sum_{n=1}^{\infty} \frac{x}{n^2+x}$, $0 \leq x \leq 1$ can be integrated term by term.

- e. Show that $\{f_n\}$ where $f_n(x) = \frac{x}{1+nx^2}$ converges uniformly on $[0,1]$ and

hence show that $\lim_{n \rightarrow \infty} \frac{1-nx^2}{(1+nx^2)^2} = 0$

- f. A retail shop attended by a single man has average of four customers an hour who buy retail items. The man attends to one customer at a time. It takes him six minutes, on the average to attend to one customer. Assuming that the arrivals are Poisson and service time has the exponential distribution. You are required to:

- (a) Find the proportion of time during which the shop is empty.
- (b) Find the probability of finding at least one customer in the shop?
- (c) What is the average number of customers in the system?
- (d) Find the average time spent, including service.
- g. A company has three factories & four customers. The company furnishes the following schedule of loss per unit on transportation of its goods to the customers in Rupees.

Factory	Customers			SUPPLY
	A	B	C	
P	4	19	22	50
Q	0	9	14	30
R	6	6	16	70
DEMAND	40	20	60	

Find the optimum cost of transpotation.

- h. Solve the following LPP using Big-M method.

Minimize , $Z = 2x_1 + x_2$

Subject to $x_1 + x_2 \geq 4;$

$2x_1 - x_2 \geq 3;$

$x_1 \geq 0, x_2 \geq 0$