Carmel College Arts Science and Commerce for Women, Nuvem Goa
B.Sc. CBCS Semester IV Regular Examination July 2021

Subject: Mathematics

Paper name and code: Analysis and Operations research (DSC 1D)

Duration: 2hr **Total marks: 40** **total number of pages: 03**

Instructions: *i) All questions are compulsory*
ii) Figures to the right indicate full marks.

Q1. Answer <u>ANY FIVE</u> of the following: **(5 X 2 mks)**

a. When is a series said to be absolutely convergent? Does $\sum_{n=1}^{\infty} \frac{(-1)^{n+2}}{n^2}$ converge absolutely.

b. Does $\sum_{n=2}^{\infty} \frac{1}{n^3 \, logn}$ converge?

c. State True of False. Justify your answer.

Every convergent series is absolutely convergent.

d. Show that if f is integrable on $[a, b]$ then f^2 is integrable on $[a, b]$.

e. State True of False. Justify your answer.

Every bounded function on [a,b] is Riemann integrable on [a,b].

f. Find the pointwise limit of the $\langle f_n \rangle$ defined by $f_n(x) = \frac{x^n}{1+x^n}$ $x \in [0,2]$

g. A certain item has a uniform demand rate of 24,000 units per year. The demand is fixed and shortages are not allowed. The inventory holding cost is Rs.0.40 per unit per month and the set-up cost per run is Rs. 300/- per run.

Determine (a) the optimal lot size, (b) Optimum time between production runs.

h. Find the dual of: Minimize , $Z = x_1 - 3x_2 - 2x_3$

Subject to $3x_1 - x_2 + 2x_3 \leq 7;$

$2x_1 - 4x_2 = 12;$

$3x_1 + 3x_2 + 8x_3 \geq 10;$

$x_1 \geq 0,\ x_2 \geq 0,\ x_3$ unrestricted

Q2. Answer <u>ANY SIX</u> of the following: (4 X 5 mks)

a. Test for convergence the following series:

i) $\sum_{n=1}^{\infty} e^{-n^2}$

ii) $\sum_{n=}^{\infty} \frac{3^{1-2n}}{n^2+1}$

b. Let f and g be differentiable functions with f ' and g' as Riemann integrable on [a, b] then prove that

$$\int_a^b f(x)g'(x)dx + \int_a^b f'(x)g(x)dx = f(b)g(b) - f(a)g(a).$$

c. Determine two partitions P and Q of the interval [0,1] such that Q is the refinement of P. Given $f: [0,1] \to R,\ f(x) = x^3$, find the upper Riemann sum and the lower Riemann sum corresponding to each of the partitions P and Q. Hence show that $L(P, f) \leq L(Q, f)$ and $U(Q, f) \leq U(P, f)$.

d. Show that $\sum_{n=1}^{\infty} \frac{x}{n^2+x}$, $0 \leq x \leq 1$ can be integrated term by term.

e. Show that $\{f_n\}$ where $f_n(x) = \frac{x}{1+nx^2}$ converges uniformly on [0,1] and hence show that $\lim\limits_{n\to\infty} \frac{1-nx^2}{(1+nx^2)^2} = 0$

f. A retail shop attended by a single man has average of four customers an hour who buy retail items. The man attends to one customer at a time. It takes him six minutes, on the average to attend to one customer. Assuming that the arrivals are Poisson and service time has the exponential distribution. You are required to:

(a) Find the proportion of time during which the shop is empty.

(b) Find the probability of finding at least one customer in the shop?

(c) What is the average number of customers in the system?

(d) Find the average time spent, including service.

g. A company has three factories & four customers. The company furnishes the following schedule of loss per unit on transportation of its goods to the customers in Rupees.

Factory	Customers			SUPPLY
	A	B	C	
P	4	19	22	50
Q	0	9	14	30
R	6	6	16	70
DEMAND	40	20	60	

Find the optimum cost of transpotation.

h. Solve the following LPP using Big-M method.
Minimize , $Z = 2x_1 + x_2$
Subject to $x_1 + x_2 \geq 4;$
$2x_1 - x_2 \geq 3;$
$x_1 \geq 0, x_2 \geq 0$