Carmel College Arts Science and Commerce for Women, Nuvem Goa

B.Sc. CBCS Semester II Regular Examination July 2021

Subject: Mathematics

Paper name and code: Matrices and Linear Algebra (DSC 1B)

Duration: 2hr   Total marks: 40   Total number of pages: 03

Instructions: *i) All questions are compulsory*

ii) Figures to the right indicate full marks.

Q1. Answer <u>ANY FIVE</u> of the following: **(5 X 2mks)**

a. Let {u, v, w} be a linearly independent set in a vector space V. Use the definition of linear independence to give a proof that the set {u + v, u + w, v + w} is linearly independent in V.

b. Let T : $R^2 \rightarrow R^2$ be the linear transformation such that T(1, 1) = (0, 2) and T(1, −1) = (2, 0)

  i. Compute T(-2, 1)

  ii. Find the matrix of T.

c. Find the matrix A whose eigenvalues are $\lambda_1 = -1$, $\lambda_2 = 1/2$, and $\lambda_3 = 1/3$, and whose corresponding eigenvectors are

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, e_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, e_3 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

d. Suppose V is a real inner product space.

  i. Show that $\langle u + v , u - v \rangle = \|u\|^2 - \|v\|^2$. For every u, v in V.

  ii. Show that if u, v ∈ V have the same norm, then u+v is orthogonal to u-v.

e. Show that the set $X = \{(x, y, z) \in \mathbb{R}^3 \mid x + y - 2z = 12 \}$ is an affine space of $\mathbb{R}^3$.

f. Reduce the following matrix to its row echelon form and hence find its rank.

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 6 \end{bmatrix}$$

g. Solve the following system of equations using Cramer's Rule

$$2x + y = 6$$

$$2x + 4y = 8$$

h. Find the eigen values of $A = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$

Q2. Answer <u>ANY SIX</u> of the following: (6 X 5mks)

a. Using the Gram-Schmidt process, transform the basis B into an orthonormal basis of R^3 where, B={ $(1,1,1),\ (-1,1,0), (1,2,1)$ }

b. Verify Cayley Hamilton theorem, hence find A^4 and A^{-1} where

$$A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

c. Let S={(x,y,z) : x+2y-z=0}

i. Show that the set S is a subspace of R^3.

ii. Find the basis of this subspace and hence determine the dimension of S.

iii. Extend the basis of S to the basis of R^3.

d. Define T : $R^4 \rightarrow R^3$ by

T (x_1, x_2, x_3, x_4) = $(2x_1 + x_3 + x_4,\ x_1 - 2x_2 - x_3, x_2 - x_3 + x_4)$

i. Show that T is a linear transformation

ii. Find the Kernel and Range of T.

iii. Determine the Nullity and Rank of T and hence verify Rank Nullity theorem.

iv. Find a vector $x \in R^4$ such that T(x) = (3, −1, −3).

e. If W is an affine space in V, then show that W is a coset of some vector subspace W' of V.

f. Find the solution of the following system of equations using Gaussian elimination if it exists.

$$x + 2y + 3z = 1$$
$$2x + 3y + z = 2$$
$$3x + 5y + 4z = 4$$

g. Find the inverse of the following matrix using elementary row reduction.

$$\begin{bmatrix} 1 & 3 & 1 \\ 5 & 6 & 3 \\ 6 & 9 & 5 \end{bmatrix}$$

h. Prove that if V is a vector space, then its additive identity is unique. That is, show that if 0 and $\tilde{0}$ are vectors in V such that x + 0 = x for all x ∈ V and x + $\tilde{0}$ = x for all x ∈ V , then 0 = $\tilde{0}$