

**CARMEL COLLEGE OF ARTS, SCIENCE & COMMERCE FOR WOMEN,
NUVEM-GOA**

B.Sc. CBCS Semester V Examination January 2021

Subject: Mathematics

Paper name and code: Calculus of 2 and 3 variables (MTC 107)

Duration: 2hrs

Total marks:80

No. of pages:2

Instructions: 1. All questions are compulsory
2. Figures to the right indicate full marks
3. Use of non-programmable calculator is allowed

I) Answer any four of the following questions. (4X4mks=16mks)

1. State Cauchy Schwarz inequality and Verify it for $\bar{x} = (1,3,0,0)$ and $\bar{y} = (-4,0,1,1)$
2. Compute the directional derivative of $f(x, y, z) = x^2 + y^2 + z^2$ at a point $(1, 2, 1)$ in the direction of $(1, 1, 0)$
3. Find the equation of the tangent plane to the surface $x^2 + 2y^2 + 3xz = 10$ at $(1, 2, \frac{1}{3})$
4. Determine the velocity and acceleration vector to the curve $r(t) = (4e^t, 6t^4, \cos t)$ at $t = 1$
5. Find $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 + 2x^2 + xy^2 + 2y^2}{x^2 + y^2}$
6. Find the critical points of $f(x, y) = x^2 + y^2 + 6x - 4y + 13$ and classify them.

II) Answer any four of the following questions. (4X4mks=16mks)

1. For any C^2 function f , Prove that $\text{Curl}(\text{grad } f) = 0$.
2. Determine whether or not the vector field $F = (y + z) \mathbf{i} + (x + z) \mathbf{j} + (x + y) \mathbf{k}$ is irrotational, and if so, find a scalar potential function for it.
3. Evaluate $\int_0^1 \int_0^{x^2} x^2 + xy - y^2 \, dy \, dx$
4. Evaluate the line integral $\int_C x^2 dx + xy dy + dz$ where $c(t) = (t, t^2, 1)$ from $t=0$ to $t=1$
5. Use triple integral to find the volume of the box $[0,1] \times [0,2] \times [1,3]$
6. Use Gauss divergence theorem to evaluate $\iint_S F \cdot dS$, where S is the surface defined by $x^2 + y^2 \leq 1, -1 \leq z \leq 1$

III) A. Answer any One of the following questions. 6mks

i. Show that the function is continuous at (0,0),

$$f(x,y) = \begin{cases} \frac{x^2y}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

ii. Show that $f(x,y) = \frac{2xy}{(x^2+y^2)^2}$ is differentiable.

Determine whether it is C^1

B. Determine the second order Taylor's series for $f(x,y) = \frac{1}{x^2+y^2+1}$, where $x_0 = 0$ and $y_0 = 0$

6mks

IV) A. Answer any One of the following questions. 6mks

i. Compute the derivative matrix of the function

$$f(x,y,z) = (-e^{xy}, x^2z, z - yx)$$

ii. Let $\vec{r} = (x, y, z)$ and $r = \sqrt{x^2 + y^2 + z^2}$.

Prove that $\nabla^2 r^n = n(n+1)r^{n-2}$

B. Maximize the function $f(x,y,z) = x + z$ subject to the constraint $x^2 + y^2 + z^2 = 1$

6mks

V) A. Answer any one of the following questions. 6mks

i. Evaluate $\int_D \int x - y \, dx dy$ where D is the triangle with vertices (0,0), (1,1) and (1,0).

ii. Find the Volume inside the ellipsoid $x^2 + y^2 + 4z^2 = 6$

B. Evaluate using cylindrical coordinates $\iiint x + y \, dx dy dz$ over the volume enclosed by $x^2 + y^2 = 4$, $z = 4$ and $z = -1$

6mks

VI) A. Answer any one of the following questions. 6mks

i. Verify Green's theorem for $f(x,y) = xy^2\hat{i} + (y+x)\hat{j}$ in the region bounded by $y = x$, $x = 2$ and $y = 0$

ii. Use Stokes's Theorem to find the value of $\iint_S f \cdot d\vec{S}$ where,

$$f(x,y,z) = xy\hat{i} + yz\hat{j} + xz\hat{k} \text{ on the surface } x+2y+z=4.$$

B. Find the work done by a force field $F = x^2\hat{i} - xy\hat{j} + \hat{k}$ in moving a particle along a straight line joining (0,0,0) and (1,1,1).

6mks
