

Carmel College of Arts, Science and Commerce for Women, Nuvem-Goa

B.Sc. (Semester I) Examination, January 2021 (CBCS)

DSC IA: CALCULUS AND NUMERICAL METHODS

Duration: 2 hours

Max marks: 40 marks

Instruction: 1) Attempt all Questions.

2) Figures to the right indicate full marks.

3) Write the appropriate question numbers before answering.

4) Students are permitted to use non programmable scientific calculator only.

Q1) Answer ANY FIVE of the following: (2x5=10 marks)

1) Find fixed points of the following function $f(x) = x^3$

2) Examine $f(x) = \frac{x^3}{3} - \frac{x^2}{2} - 2x$; for all x in \mathbb{R} , for critical points.

3) Evaluate $\lim_{x \rightarrow 1} \frac{1 + \log x - x}{1 - 2x + x^2}$

4) Give an example of a monotonically increasing sequence which is bounded above.

5) Show that the sequence $\left\{ \frac{\sin n}{n} \right\}$ converges to 0.

6) Obtain $\nabla^3 y_6$ and $\Delta^2 y_1$ for the following data.

x_i	1	1.5	2	2.5	3	3.5	4
y_i	2	2.4	2.7	2.8	3	2.6	2.1

7) Obtain the integral of the function from $x = 1$ to $x = 4$, which passes through the following points using the Trapezoidal Rule.

x	1	1.5	2	2.5	3	3.5	4
$f(x)$	2	2.4	2.7	2.8	3	2.6	2.1

- 8) Find a root of the equation $x^3 - x = 1$, correct upto 2 decimal places, using the Newton-Raphson method by taking the initial approximation as $x_0 = 1.5$.

Q2) Answer ANY SIX of the following:

(5x6=30marks)

- 1) Let E be a non-empty bounded above subset of R. If α and β are supremums of E,
Show that $\alpha = \beta$. Find the supremum and infimum (if exists) of $\left\{\frac{1}{n} + (-1)^n; n \in N\right\}$
- 2) If $y = e^{\tan^{-1} x}$, use Leibnitz rule to show that
$$(1 + x^2)y_{n+2} + [2(n + 1)x - 1]y_{n+1} + n(n + 1)y_n = 0$$
- 3) Show that $f(x) = e^{-x}$ is continuous on (0,1) and hence prove that
 $xe^x = 1$ for some $x \in (0,1)$.
- 4) Use Maclaurin series expansion to show that $e^x \sin x = x + \frac{x^2}{2} + \frac{x^3}{3} + \dots$
- 5) Define convergence of a sequence $\{a_n\}$. Show that the sequence $\{a_n\}; a_n = \frac{7}{\sqrt{n}}$ converges to 0.
- 6) Use the regular-falsi method to obtain a root of the function $3x - \cos x - 1 = 0$ correct upto 3 decimal places.
- 7) Find the first and second derivative of the function at $x = 0.1$, whose values are tabulated below.

x_i	0.1	0.2	0.3	0.4
y_i	1.10517	1.22140	1.34986	1.49182

- 8) Using Bisection method find a root of $\cos x - xe^x = 0$ between 0 and 1 correct upto 3 decimal places.
