

**Carmel College Arts Science and Commerce for Women, Nuvem Goa**  
**B.Sc. CBCS Semester VI Regular Examination July 2021**

**Subject: Mathematics**

**Paper name and code: Differential Equations II (MTC 108)**

**Duration: 2hr**

**Total marks: 30**

**Total number of pages: 02**

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**Instructions:** i) All questions are compulsory  
ii) Figures to the right indicate full marks.

**Q1. Answer ANY FOUR of the following:**

**(5 X 2 mks)**

- a. Find the inverse Laplace transform of  $F(s) = \frac{1+e^{-2s}}{s^2+6}$
- b. Use Laplace transforms to evaluate  $\int_0^\infty \frac{\sin x}{x} dx$
- c. Use Euler's method to estimate the solution of  $y' = y - \sin x$  at  $x=1$ . Given  $y(0) = -1$ , take  $h=0.05$ .
- d. Given  $\frac{dy}{dx} = x^3 + y$ ,  $y(0) = 2$ . The values of  $y(0.2) = 2.073$ ,  $y(0.4) = 2.452$ , and  $y(0.6) = 3.023$  are gotten by the R.K. method of the order. Find  $y(0.8)$  by Milne's predictor-corrector method taking  $h = 0.2$ .
- e. Find the power series solution to the differential equation.
- $y' = 2xy$
- f. Prove that  $J_n(x) = \frac{x}{2n} [J_{n-1}(x) + J_{n+1}(x)]$
- g. Solve the differential equation
- $x^2 y' = x^2 + xy + y^2$
- h. Solve the differential equation

$$\frac{dy}{dx} = \frac{(x+y) + (x+y-1)\log(x+y)}{\log(x+y)}$$

**Q2. Answer ANY FOUR of the following:**

**(4 X 5 mks)**

- a. Use Frobenius method to solve  $y'' + xy' + x^2 y = 0$  near  $x=0$ .

b. Prove that  $\int_{-1}^1 x P_n(x) P_{n-1}(x) dx = \frac{2n}{4n^2-1}$

c. Use Convolution theorem to find  $L^{-1} \left[ \frac{s+3}{(s^2+6s+13)^2} \right]$

d. Use forth order Runge Kutta method to find y at x=0.2 given that

$$\frac{dy}{dx} = 3e^x + 2y, y(0)=0 \text{ and } h=0.1.$$

e. Use method of Variation of parameters to solve

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \log x$$

f. Use method of undetermined coefficients to solve

$$(D^2 - 2D + 3)y = x^3 + \cos x$$

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