CARMEL COLLEGE OF ARTS, SCIENCE AND COMMERCE FOR WOMEN, NUVEM-GOA

B.Sc. CBCS Semester V Examination, January, 2021

Subject Code: (MTE101) **Subject Name: Foundation of Mathematics**

Total marks: 80 **Duration: 2 Hrs** **Total No. of pages: 2**

Instructions: *1. All questions are compulsory, however internal choice is available.*
2. Figures to the right indicate maximum marks allotted to the question.
3. Use of Scientific non-programmable calculator is allowed.

Q.1. Attempt **any four** of the following: **[16]**

a) "Let A, B be two set, we write A subset of B if every element of A is also an element of B". Write this in symbols using quantifiers, negate it and write the negation in words.

b) Given the statement, "For an integer m, if m is odd, then m^2 is not divisible by 4." Identify the statement, hence write the converse, inverse and the contrapositive of the statement.

c) Identify the set $A = \left\{x \in \mathbb{R} : \ x + \frac{1}{x} \geq 2\right\}$.

d) Show that $f: [0, \infty) \to [0,1)$, given by $f(x) = {x^2}/{1 + x^2}$ is a bijection and find its inverse.

e) Prove the following statement by contradiction,
"For a real number $a \geq 0$, if for each $\varepsilon > 0$, we have $a \leq \varepsilon$, then $a = 0$."

f) For any three sets A, B, C, Show that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Q.2. Attempt **any four** of the following: **[16]**

a) Using Induction Principle, prove that for each $n \in \mathbb{N}$, $n(n+1)(n+2)(n+3)$ is divisible by 24.

b) For $n \in \mathbb{N}$, define a_n as follows: $a_1 = 1, a_2 = 8$ and $a_n = a_{n-1} + 2a_{n-2}$ for $n \geq 3$. Prove that for each $n \in \mathbb{N}$, $a_n = 3.\,2^{n-1} + 2(-1)^n$.

c) State and prove the Cantor's theorem.

d) Use the Well- Ordering Principle to show that any infinite subset of $\mathbb{N}$ is countably infinite.

e) Let A be a subset of a partially ordered set $(X, \leq)$. Prove that the LUB of a set, if it exists is unique.

f) A be a subset of a partially ordered set $(X, \leq)$, define the following terms:

i. Minimal element of A.
ii. Lower bound
iii. Greatest lower bound

Q.3.A. Attempt **any one** of the following: **[6]**

a. Prove the statement, "There is no rational number x, such that $x^2 = 15$."

b. Find whether the function $f: [0, 2\pi) \to D = \{(x, y): x^2 + y^2 = 1\}$,

$f(x) = (\cos x, \sin x)$ is a bijection.

B. For any set P and Q, show that the following statements are equivalent:

i. $P \subseteq Q$ ii. $P \cap Q = P$ iii. $P \cup Q = Q$ **[6]**

Q.4.A. Attempt **any one** of the following: **[6]**

a. Let $f: X \to Y$ and $g: Y \to Z$ be functions, show that

i. If f and g are injective, then $g \circ f$ is injective.

ii. If f and g are surjective, the $g \circ f$ is surjective.

iii. If f and g are bijections, then $g \circ f$ is a bijection.

b. A function $f: \mathrm{X} \to \mathrm{Y}$ is one-one if and only if $f(A \cap B) = f(A) \cap f(B)$ holds for all subsets A and B of X.

B. A relation $\sim$ on $\mathbb{Z} \times \mathbb{Z}^*$, where $\mathbb{Z}^* = \mathbb{Z} \setminus \{0\}$ is given by $(a, b) \sim (c, d)$ if and only if $ad = bc$. Show that $\sim$ is an equivalence relation. Hence, find the equivalence classes and a suitable transversal. **[6]**

Q.5.A. Attempt **any one** of the following: **[6]**

a. Let a, b be integers, not both zero, and let d be the greatest common divisor of a and b. Use the Well-Ordering Principle and show that there exist integers x, y such that $d = ax + by$.

b. Using the Induction Principle, prove that for each $n \in \mathbb{N}$,

$$1.n + 2(n-1) + 3(n-2) + \ldots. + n.1 = \frac{n(n+1)(n+2)}{6}$$

B. State and prove the Schröeder-Berstain Theorem. **[6]**

Q.6.A. Attempt **any one** of the following: **[6]**

a. Prove that if $f: A \to I_n$ is one-one, then A is finite and $|A| \leq n$

b. On $\mathbb{C}$, define $z_1 R z_2$, if and only if $|z_1| \leq |z_2|$. Is this a partial order on $\mathbb{C}$? Justify. (where for $z = x + iy$, $|z| = \sqrt{x^2 + y^2}$)

B. For a non-empty set A, show that the following are equivalent. **[6]**

i. A is countable.

ii. There is a one-one map of A into $\mathbb{N}$.

iii. There is an onto map from $\mathbb{N}$ onto A.
