CARMEL COLLEGE OF ARTS, SCIENCE AND COMMERCE FOR WOMEN, NUVEM-GOA
SEMESTER END EXAMINATION AUGUST 2020

Semester: VI OF B.SC   **Course name & Code: Complex Analysis (MTC109)**
Total marks: 30   **Date: 03/08/2020**   **Duration: 2 Hrs**   **Total No. of pages: 02**

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Instructions:

1. *All questions are compulsory, however internal choice is available.*
2. *Figures to the right indicate maximum marks allotted to the question.*
3. *Student shall write down the answers and should sign each and every page with date and then upload the scanned copy/photograph of the answer sheet in PDF format. A student must upload their answer scripts by 2.00 pm.*
4. *PDF should be titled as : Name of the student, Seat Number and paper name.*

Q.1. Attempt **any five** of the following: **[10]**

    a) Find the Principal argument $Arg\ z$, when $z = (\sqrt{3} - i)^6$.

    b) Given the case $(-16)^{1/4}$, find all the roots in rectangular coordinates.

    c) Find the $\lim\limits_{z \to -2i} \dfrac{(2z+3)(z-1)}{z^2-2z+4}$.

    d) Use the definition of derivative to find the derivative of f(z)=$z^3 - 2z$ at the point $z = z_0$.

    e) Find all the values of z such that $e^z = 1 + \sqrt{3}i$

    f) Evaluate the integrals $\int_c \dfrac{1}{z}dz$, where c is the semi-circle, $z = e^{i\theta}, -\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$.

    g) Evaluate the integral $\int_1^2 \left(\dfrac{1}{t} - ti\right)dt$.

    h) Expand $f(z) = ze^{2z}$ in a Taylors series about $z = -1$

Q.2. Attempt **any four** of the following: **[20]**

    a) When do you say a function $f(z) = u(x,y) + i\ v(x,y)$ is analytic? Check the analyticity of the following functions and find their derivatives.
       i) $f(z) = e^{-z}$
       ii) $f(z) = \cos z$

    b) Prove that $u = 2x - x^3 + 3xy^2$ is a harmonic function. Determine its harmonic conjugate and hence find its corresponding analytic function.

    c) Find the principal value of $(1 - i)^{4i}$

    d) Find the value of the integral $\int_c \dfrac{3z^3+2}{(z-1)(z^2+9)}dz$ taken counterclockwise around the circle $|z| = 4$

    e) Find the Laurent series expansion in powers of z for the function $f(z) = \dfrac{z}{(z-1)(z-3)}$ valid in the region $0 < |z - 1| < 2$

f) Use the Residue theorem to calculate the total residue of the following functions at the poles

    i. $f(z) = \dfrac{z}{z^2+2z+5}$ ii. $f(z) = \dfrac{2z^2}{(z+1)(z-2)}$

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