CARMEL COLLEGE OF ARTS, SCIENCE AND COMMERCE FOR WOMEN, NUVEM-GOA

B.Sc. CBCS Semester VI Examination, July, 2021

Subject Code: (MTC109) **Subject Name: Complex Analysis**

Total marks: 30 **Date: 17-07-2021** **Duration: 2 Hr** **Total No. of pages: 2**

Instructions:

1. All questions are compulsory, however internal choice is available.

2. Figures to the right indicate maximum marks allotted to the question.

*3. Student shall write down the answers and should **sign each and every page with date** and then upload the scanned copy/photograph of the answer sheet in PDF format. A student must upload their answer scripts by 1.00 pm.*

*4. PDF should be titled as : **Seat Number_ Name of the student _Paper code.***

Q.1. Attempt **any five** of the following: **[10]**

a) Reduce the quantity to a real number.
$$\frac{10i}{(2-2i)(2-i)(3-i)}$$

b) Use the exponential form of complex number to evaluate
$$\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^{10}$$

c) Define the Complex exponent function z^c.
Use the definition of z^c to show that $\left(-1+\sqrt{3}i\right)^{3/2} = \pm 2\sqrt{2}$

d) Evaluate using the theorems on limits.
$$\lim_{z \to i/2} \frac{(2z-3)(4z+i)}{(iz-1)^2}$$

e) Find the residue of the following function at the given point.
$$f(z) = \frac{z^2}{(z-2)^2(z^2+9)} \quad \text{at } z = 2$$

f) Evaluate
$$\int_C \frac{z+2}{z} dz \; ; \quad \text{where } C: semicircle\ z = 2e^{it} \left(\frac{\pi}{2} \le t \le \frac{3\pi}{2}\right)$$

g) Find the Laurent series $f(z) = \frac{1}{z^5(1-z)}$ valid in the region $|z| > 1$

h) Find the value of the integral of $g(z) = \frac{1}{z^2+4}$ around the circle $|z-i| = 2$ in the positive sense. State the result used.

Q.2. Attempt **any four** of the following: **[20]**

a) Determine whether following functions are analytic
(i) $f(z) = i(\overline{z})^2$ (ii) $f(z) = \exp \overline{z}$

b) For a complex number z define the function $\log z$ and hence prove the following properties.
1. $\log(z_1 z_2) = \log z_1 + \log z_2$
2. $\log\left(\frac{z_1}{z_2}\right) = \log z_1 - \log z_2$
3. $\log z^n = n \log z$; where z_1 and z_2 are complex numbers.

c) Evaluate the integral of $f(z) = \frac{(3z+2)^2}{z(z-1)(2z+5)}$ around the positively oriented circle $|z| = 4$

d) Obtain the Taylor's series expansion of $f(z) = \frac{z+3}{z^2-5z+4}$ about the point $z = 2$.

e) Solve the equation $\cos z = \sqrt{2}$ for z

f) Find all the zeros and poles of $f(z) = \frac{z^2+4}{z^3+2z^2+2z}$ and determine the residues at the poles.

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