

**Carmel College of Arts, Science and Commerce for Women,
Nuvem – Goa.**

July 2021

Semester – V of B.Sc Course Name: Combinatorics
Total Marks: 30 Date: 28/07/2021 Duration: 2 hours Total number of pages : 1

I. Answer any five of the following

(2 x 10 = 20)

- 1) A busy airport sees 1500 takeoffs per day. Prove that there are at least two planes that takeoff within a minute of each other.
- 2) Prove that the sequence 1967, 19671967, 196719671967, ... contains an element that is divisible by 1969.
- 3) Prove for all positive integers n that $1 + 3 + 5 + \dots + (2n - 1) = n^2$ by using induction.
- 4) How many compositions does 13 have whose first part is not 1?
- 5) How many partitions of the set $\{1,2,3,4,5,6\}$ contains the singleton block $\{1\}$.
- 6) Compute the values of $S(4,k)$ for $k = 1,2,3,4$
- 7) Give a combinatorial proof for the following identity.

$$\binom{n}{k} = \binom{n}{n-k} \text{ for } k \leq n$$

- 8) Give a combinatorial proof for the following identity.

$$\sum_{k=1}^n k \binom{n}{k} = n 2^{n-1}$$

II. Answer any four of the following

(4 x 5 = 20)

- 1) In how many ways can the elements of $[n]$ be permuted if 1 is to precede 2 and 3 is to precede 4?
- 2) Show that $S(n,k) = S(n-1,k-1) + n S(n-1, k)$.
- 3) A certain kind of insect population multiplies so that at the end of each year, its size is the double of its size a year before plus 1000 more insects. Assuming that originally 50 insects were released, how many of them will we have at the end of 70th year?
- 4) Show that the number of subsets of $[n]$ is 2^n .
- 5) A semester at a technical university consists of n days. At the beginning of each semester, the Dean of Engineering designs the term in the following way. She splits the term into two parts. The first k days of the term will form the theoretical part of the semester, and the second $n - k$ days will form the laboratory part (here $1 \leq k \leq n - 2$). Then she chooses one holiday in the first part, and two holidays in the second part.
In how many different ways can she design the term with these constraints?
- 6) Show that the number of partitions of n into distinct odd parts is equal to that of all self-conjugate partitions of n .
