CARMEL COLLEGE OF ARTS, SCIENCE AND COMMERCE FOR WOMEN.
SEMESTER END EXAMINATION, AUGUST 2020.

SEMESTER : VI of B.SC.	DATE : 07/08/2020	TOTAL MARKS: 30
SUBJECT : MATHEMATICS	PAPER NAME AND PAPER CODE : Number Theory (MTE 103)	DURATION : 10 a.m. -12noon

No. OF PAGES: 01

Instructions:

1. All Questions are Compulsory, however internal choice is available.
2. Figures to the right indicate full marks allotted to questions/sub questions.
3. Use of Non-programmable calculator is allowed.
4. Student shall write down the answers and should sign each and every page with date and then upload the scanned copy/photograph of the answer sheet in PDF format. A student must upload their answer scripts by 2.00 pm.
5. PDF should be titled as: Name of the student, Seat Number and paper name.

Q1. Answer any <u>five</u> of the following: **2x5=10**

a) Let gcd(a,b) = 1, then prove that gcd(a+b, a – b) = 1 or 2.
b) Find all the solutions in positive integers of the linear Diophantine equation $54x + 21y = 906$.
c) Find two Pythagorean triple not necessarily primitive of the form 16, y, z .
d) Find the remainder when $1^5 + 2^5 + \ldots + 100^5$ is divided by 4.
e) Give an example to show that $a^2 \equiv b^2 (\text{mod } n)$ need not imply that $a \equiv b (\text{mod } n)$.
f) Find out how many times the factor 3 appears in 40!
g) Prove that for an integer n > 1, if $\tau(n)$ is odd then n is a perfect square.

Q2. Answer any <u>four</u> of the following: **5x4= 20**

a) Prove that an integer n > 1 is prime iff $(n - 2)! = 1(\text{mod } n)$.
b) State and prove the conditions and the formulae to obtain the solutions of Pythagorean equation $x^2 + y^2 = z^2$.
c) Let a, b, c $\in \mathbb{Z}$. Prove that gcd(a,b,c) = gcd(a, gcd(b,c)).
d) Solve the following system of equations using Chinese remainder theorem
$x \equiv 1(\text{mod } 6)$; $x \equiv 5(\text{mod } 7)$; $x \equiv 2(\text{mod } 11)$
e) For a prime p of the form 4k + 3, prove that either ;
$\left(\frac{p-1}{2}\right)! \equiv 1(\text{mod } p)$ *or* $\left(\frac{p-1}{2}\right)! \equiv -1(\text{mod } p)$; hence $\left(\frac{p-1}{2}\right)!$ satisfy the quadratic congruence $x^2 \equiv 1(\text{mod } p)$.
f) Prove that τ and σ are multiplicative functions.

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