

**Carmel College of Arts Science and Commerce for Women,
Nuvem-Goa**

Semester End Examination August 2020

Semester: V of B.Sc. Course name and Code : Combinatorics (MTE 102)

Total marks: 30 Date:11/08/2020 Duration: 2hours Total no. of pages: 1

Instructions: i) All questions are compulsory
ii) Figures to the right indicate full marks
iii) Use of non-programmable calculators is allowed.
iv) Student shall write down the answers and should sign each and every page with date and then upload the scanned copy/photograph of the answer sheet in PDF format. A student must upload their answer scripts by 2.00 pm.
v) PDF should be titled as: Name of the student, Seat Number and paper name.

I) Answer any five of the following. (2x5 = 10)

- 1) Given $n + 1$ integers from the set $\{1, 2, 3, \dots, 2n\}$, show that there will always be two among the selected integers whose greatest common divisor is 1.
- 2) Prove that there exists a positive integer n such that $44^n - 1$ is divisible by 7.
- 3) Give a combinatorial proof for the following identity where n, k, m are non-negative integers such that $k + m \leq n$.
$$\binom{n}{m} \binom{n-m}{k} = \binom{n}{k} \binom{n-k}{m}$$
- 4) Let the sequence $\{a_n\}$ be such that $a_0 = 0$ and $a_{n+1} = a_0 + a_1 + a_2 + \dots + a_n + n + 1 \forall n \in \mathbb{N}$. Show that the equality $a_n = 2^n - 1$ holds $\forall n$.
- 5) How many compositions does the integer 15 have whose first part is not 1?
- 6) How many partitions of the set $\{1, 2, 3, 4, 5\}$ contain at least one of the singleton blocks $\{1\}$ and $\{5\}$?
- 7) What is the number of partitions of $[8]$ into two blocks in which the two blocks do not have the same size?
- 8) Compute the values of $c(5, k)$ for $k = 1, 2, 3, 4, 5$.

II) Answer any four of the following questions. (5x4 = 20)

- 1) Find an explicit formula for a_n if $a_0 = 1$, $a_{n+1} = 3a_n + 2^n$, if $n \geq 0$.
- 2) How many 3-digit numbers are there in which the sum of the digits is even?
- 3) All n soldiers of a military squadron stand in a line. The officer in-charge splits the line at several places forming smaller non-empty units. Then he names one person in each unit to be the commander of that unit. Let h_n be the number of ways he can do this. Find a closed formula for h_n .
- 4) A certain kind of insect population multiplies so that at the end of each year, its size is the double of its size a year before, plus 1000 more insects. Assuming that originally we released 50 insects, how many of them will we have at the end of the 70th year?
- 5) Let i and j be two elements of n . Then show that i and j are in the same cycle in exactly half of all permutations.
- 6) Find a closed formula for $S(n, 2)$. Use this to obtain a formula for $S(n, 3)$.
