CARMEL COLLEGE OF ARTS, SCIENCE AND COMMERCE FOR WOMEN.
SEMESTER END EXAMINATION, AUGUST 2020.

SEMESTER : V(Repeat) of B.SC.	**DATE : 13/08/2020**	**TOTAL MARKS: 30**
SUBJECT : MATHEMATICS	**PAPER NAME AND PAPER CODE : Analysis II (MTC 106)**	**DURATION : 10 a.m. -12noon**

No. OF PAGES: 01

Instructions:

1. All Questions are Compulsory, however internal choice is available.
2. Figures to the right indicate full marks allotted to questions/sub questions.
3. Use of Non-programmable calculator is allowed.
4. Student shall write down the answers and should sign each and every page with date and then upload the scanned copy/photograph of the answer sheet in PDF format. A student must upload their answer scripts by 2.00 pm.
5. PDF should be titled as: Name of the student, Seat Number and paper name.

Q1. Answer any <u>five</u> of the following: 2x5=10

1. Examine the convergence of the improper integral $\int_1^\infty \frac{\sin^2 x}{x^2} dx$.
2. Use comparison test in limit form to discuss the convergence of $\int_0^1 \frac{1}{x(1+x)} dx$.
3. Prove that $\Gamma(n+1)=n!$ where Γ denotes gamma function .
4. Find the radius of convergence and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{n!}$.
5. If the real valued functions f, g, h given by f(x)=1, g(x)=x and h(x)=1+Ax+Bx^2 are orthogonal functions w.r.t usual integral inner product on C[-1,1] find the value of A and B.
6. Find the Fourier series of f(x)=x on [-π, π]
7. Find the Fourier cosine series of f(x)= $x - x^2; x$ € [0,1]
8. Prove that every orthogonal set of non-zero vectors is linearly independent in the inner product space.

Q2A. Answer any <u>four</u> of the following: 5x4=20

1. Let Ø= {$Ø_1$, $Ø_2$, $Ø_3$. . . . } be an orthonormal set in C[a, b] with respect to the usual integral inner product on C[a, b]. If $f \in C[a,b]$ *and if* $c_k = \langle f, Ø_k \rangle$ then prove that $\sum_{k=1}^{\infty} c_k^2 \leq ||f||^2$.
2. Find the fourier series of f(x)=x^2, x € [- π, π] and deduce $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots. = \frac{\pi^2}{8}$

3. Show that $S = \{1, \text{Sinnx}, \text{Cosnx}\}_{n=1}^{\infty}$ an orthogonal set in C[-π, π] with usual integral inner product.

4. Using the properties of beta function show that $\int_0^{\pi} \sin^7\theta.\cos^7\theta \, d\theta = 0$.

5. Discuss the convergence of $\int_0^{\infty} \frac{x^p}{(1+x^2)^q} dx$ where p and q are real number.

6. For any positive integer m prove that

$$2^{2m-1}\Gamma(m)\Gamma(m+\frac{1}{2}) = \Gamma(\pi)\Gamma(2m) \text{ where } \Gamma \text{ represents gamma function.}$$

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