

**Carmel College of Arts Science and Commerce for Women,  
Nuvem-Goa**

**Semester End Examination August 2020**

**Semester: V of B.Sc.      Course name and Code : Combinatorics (MTE 102)**

**Total marks: 30    Date:11/08/2020    Duration: 2hours    Total no. of pages: 1**

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- Instructions:** i) All questions are compulsory  
ii) Figures to the right indicate full marks  
iii) Use of non-programmable calculators is allowed.  
iv) Student shall write down the answers and should sign each and every page with date and then upload the scanned copy/photograph of the answer sheet in PDF format. A student must upload their answer scripts by 2.00 pm.  
v) PDF should be titled as: Name of the student, Seat Number and paper name.

**I) Answer any five of the following. (2x5 = 10)**

- 1) Given  $n + 1$  integers from the set  $\{1,2,3, \dots, 2n\}$ , show that there will always be two among the selected integers whose greatest common divisor is 1.
- 2) Prove that there exists a positive integer  $n$  such that  $44^n - 1$  is divisible by 7.
- 3) Give a combinatorial proof for the following identity where  $n, k, m$  are non-negative integers such that  $k + m \leq n$ .
$$\binom{n}{m} \binom{n-m}{k} = \binom{n}{k} \binom{n-k}{m}$$
- 4) Let the sequence  $\{a_n\}$  be such that  $a_0 = 0$  and  $a_{n+1} = a_0 + a_1 + a_2 + \dots + a_n + n + 1 \forall n \in \mathbb{N}$ . Show that the equality  $a_n = 2^n - 1$  holds  $\forall n$ .
- 5) How many compositions does the integer 15 have whose first part is not 1?
- 6) How many partitions of the set  $\{1,2,3,4,5\}$  contain at least one of the singleton blocks  $\{1\}$  and  $\{5\}$ ?
- 7) What is the number of partitions of  $[8]$  into two blocks in which the two blocks do not have the same size?
- 8) Compute the values of  $c(5, k)$  for  $k = 1,2,3,4,5$ .

**II) Answer any four of the following questions. (5x4 = 20)**

- 1) Find an explicit formula for  $a_n$  if  $a_0 = 1, a_{n+1} = 3a_n + 2^n$ , if  $n \geq 0$ .
- 2) How many 3-digit numbers are there in which the sum of the digits is even?
- 3) All  $n$  soldiers of a military squadron stand in a line. The officer in-charge splits the line at several places forming smaller non-empty units. Then he names one person in each unit to be the commander of that unit. Let  $h_n$  be the number of ways he can do this. Find a closed formula for  $h_n$ .
- 4) A certain kind of insect population multiplies so that at the end of each year, its size is the double of its size a year before, plus 1000 more insects. Assuming that originally we released 50 insects, how many of them will we have at the end of the 70th year?
- 5) Let  $i$  and  $j$  be two elements of  $n$ . Then show that  $i$  and  $j$  are in the same cycle in exactly half of all permutations.
- 6) Find a closed formula for  $S(n, 2)$ . Use this to obtain a formula for  $S(n, 3)$ .

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