CARMEL COLLEGE OF ARTS, SCIENCE AND COMMERCE FOR WOMEN.
SEMESTER END EXAMINATION, AUGUST 2020.

SEMESTER: VI of B.SC.	DATE : 05/08/2020	TOTAL MARKS: 30
SUBJECT : MATHEMATICS	PAPER NAME AND PAPER CODE : Metric Spaces (MTC 110)	DURATION : 10 a.m. -12noon

No. OF PAGES: 01

Instructions:

1. All Questions are Compulsory, however internal choice is available.
2. Figures to the right indicate full marks allotted to questions/sub questions.
3. Use of Non-programmable calculator is allowed.
4. Student shall write down the answers and should sign each and every page with date and then upload the scanned copy/photograph of the answer sheet in PDF format. A student must upload their answer scripts by 2.00 pm.
5. PDF should be titled as: Name of the student, Seat Number and paper name.

Q1. Answer any <u>five</u> of the following: **2x5=10**

a) Check if $d:\mathbb{R}^2\times\mathbb{R}^2\to\mathbb{R}$ defined as d(x, y) = $|x_1+y_1|-|x_2+y_2|$ is a metric space on $\mathbb{R}^2$.

b) Prove that every convergent sequence in a metric space has a unique limit point.

c) Prove that in any metric space compliment of every singleton set is open.

d) In a metric space, Every Cauchy sequence is convergent. Prove or Disprove.

e) Let (X,d) be a metric space and let f: X→X be a continuous function. Show that the set W={x∈X/ f(x) = x}is closed in X.

f) Let $(\mathbb{R}^2, d_u)$ be a metric space with usual metric on $\mathbb{R}^2$. Show that $f:\mathbb{R}^2\to\mathbb{R}^2$ given by f((x,y)) = $(\frac{x}{2},\frac{y}{2})$ ∀ (x,y) ∈ $\mathbb{R}^2$ is a contraction mapping.

g) Prove that two separated sets in a metric space (X, d) are disjoint. Is the converse true? Justify.

h) Is the set A = {(x,y) ∈ $\mathbb{R}^2$ / x ≠ 9} disconnected in $\mathbb{R}^2$ with respect to usual metric? Justify your answer.

Q2A. Answer any <u>four</u> of the following: **5x4=20**

a) Prove that S = {(x_1, x_2)∈ $\mathbb{R}^2$ / x_1 + x_2 > 1} is an open set on $\mathbb{R}^2$.

b) Give an example of a set $X\subseteq\mathbb{R}$ which contains a subset different from ϕ *and* X that is both open as well as closed in X.

c) Prove that $\mathbb{R}^k$ is a complete metric space with respect to d_∞ metric where d_∞ = Max{$|x_i-y_i|$/1≤ i ≤ k }.

d) Let (X, d) be discrete metric space and (Y, e) be any metric space then show that any function f :X→Y is continuous.

e) Prove that a discrete metric space (X,d) with more than one point is disconnected.

f) Use Picccard's theorem to prove that the differential equation $\frac{dy}{dx}=x+y+1$ has unique solution in the neighbourhood of (1, 0) which satisfy the initial condition y(1) = 4.

****