

**CARMEL COLLEGE OF ARTS, SCIENCE AND COMMERCE FOR WOMEN,  
NUVEM-GOA**

**SEMESTER END EXAMINATION AUGUST 2020**

**Semester: VI OF B.SC      Course name & Code: Complex Analysis (MTC109)**  
**Total marks: 30      Date: 03/08/2020      Duration: 2 Hrs      Total No. of pages: 02**

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**Instructions:**

1. All questions are compulsory, however internal choice is available.
2. Figures to the right indicate maximum marks allotted to the question.
3. Student shall write down the answers and should sign each and every page with date and then upload the scanned copy/photograph of the answer sheet in PDF format. A student must upload their answer scripts by 2.00 pm.
4. PDF should be titled as : Name of the student, Seat Number and paper name.

**Q.1. Attempt any five of the following: [10]**

- a) Find the Principal argument  $\text{Arg } z$ , when  $z = (\sqrt{3} - i)^6$ .
- b) Given the case  $(-16)^{1/4}$ , find all the roots in rectangular coordinates.
- c) Find the  $\lim_{z \rightarrow -2i} \frac{(2z+3)(z-1)}{z^2-2z+4}$ .
- d) Use the definition of derivative to find the derivative of  $f(z)=z^3 - 2z$  at the point  $z = z_0$ .
- e) Find all the values of  $z$  such that  $e^z = 1 + \sqrt{3}i$
- f) Evaluate the integrals  $\int_c \frac{1}{z} dz$ , where  $c$  is the semi-circle,  $z = e^{i\theta}$ ,  $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ .
- g) Evaluate the integral  $\int_1^2 \left( \frac{1}{t} - ti \right) dt$ .
- h) Expand  $f(z) = ze^{2z}$  in a Taylors series about  $z = -1$

**Q.2. Attempt any four of the following: [20]**

- a) When do you say a function  $f(z) = u(x, y) + i v(x, y)$  is analytic? Check the analyticity of the following functions and find their derivatives.
  - i)  $f(z) = e^{-z}$
  - ii)  $f(z) = \cos z$
- b) Prove that  $u = 2x - x^3 + 3xy^2$  is a harmonic function. Determine its harmonic conjugate and hence find its corresponding analytic function.
- c) Find the principal value of  $(1 - i)^{4i}$
- d) Find the value of the integral  $\int_c \frac{3z^3+2}{(z-1)(z^2+9)} dz$  taken counterclockwise around the circle  $|z| = 4$
- e) Find the Laurent series expansion in powers of  $z$  for the function  $f(z) = \frac{z}{(z-1)(z-3)}$  valid in the region  $0 < |z - 1| < 2$

f) Use the Residue theorem to calculate the total residue of the following functions at the poles

i.  $f(z) = \frac{z}{z^2+2z+5}$

ii.  $f(z) = \frac{2z^2}{(z+1)(z-2)}$

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