CARMEL COLLEGE OF ARTS, SCIENCE AND COMMERCE FOR WOMEN, NUVEM-GOA

SEMESTER END REPEAT EXAMINATION, AUGUST 2020

Semester: V OF B.SC Course name & Code: Foundation of Mathematics (MTE101)
Total marks: 30 Date:08/08/2020 Duration: 2 Hrs Total No. of pages: 2

Instructions:
1. All questions are compulsory, however internal choice is available.
2. Figures to the right indicate maximum marks allotted to the question.
3. Student shall write down the answers and should sign each and every page with date and then upload the scanned copy/photograph of the answer sheet in PDF format. A student must upload their answer scripts by 2.00 pm.
4. PDF should be titled as : Name of the student, Seat Number and paper name.

Q.1. Attempt **any five** of the following: **[10]**

a) There exists an integer $x \in \mathbb{Z}$ such that for any $y \in \mathbb{Z}$ we have $x + y = y$. Write this in symbols using quantifiers, negate it and write the negation in words.
b) Given the converse of a statement as, "For an integer m, if m is odd, then m^2 is not divisible by 4." Identify the statement, hence write its inverse.
c) Prove the following statement by contradiction, "For a real number $a \geq 0$, if for each $\varepsilon > 0$, we have $a \leq \varepsilon$, then $a = 0$."
d) Let A and B be subsets of a universal set U, then show that $(A \cup B)^c = A^c \cap B^c$.
e) Show that the function $f: \mathbb{R} \to \mathbb{R}$, $f(x) = 5x - 7$ is a bijection and find its inverse.
f) For $n \in \mathbb{N}$, define a_n as follows: $a_1 = 1, a_2 = 8$ and $a_n = a_{n-1} + 2a_{n-2}$ for $n \geq 3$. Prove that for each $n \in \mathbb{N}$, $a_n = 3.\,2^{n-1} + 2(-1)^n$.
g) On $\mathbb{C}$, define $z_1 R z_2$, if and only if $|z_1| \leq |z_2|$. Is this a partial order on $\mathbb{C}$? Justify. (where for $z = x + iy$, $|z| = \sqrt{x^2 + y^2}$)
h) Let A be a subset of a partially ordered set $(X, \leq)$. Prove that the LUB of a set, if it exists is unique

Q.2. Attempt **any four** of the following: **[20]**

a) Prove the statement, "There is no rational number x, such that $x^2 = 2$."
b) For any set P and Q, show that the following statements are equivalent:
   i. $P \subseteq Q$
   ii. $P \cap Q = P$
   iii. $P \cup Q = Q$
c) Suppose $f: X \to Y$ and $g: Y \to X$ be such that $g \circ f = Id_X$ and $f \circ g = Id_Y$. Show that f and g are bjections.

d) A relation $\sim$ on $\mathbb{Z} \times \mathbb{Z}^*$, where $\mathbb{Z}^* = \mathbb{Z} \setminus \{0\}$ is given by $(a, b) \sim (c, d)$ if and only if $ad = bc$. Show that $\sim$ is an equivalence relation. Write the quotient set and find a suitable set that can be identified with quotient set.

e) Show that the set of irrational numbers is uncountable.

f) Using the Induction Principle, prove that for each $n \in \mathbb{N}$,

$$1.n + 2(n-1) + 3(n-2) + \ \ldots. + n.1 = \frac{n(n+1)(n+2)}{6}$$

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