CARMEL COLLEGE OF ARTS, SCIENCE AND COMMERCE FOR WOMEN.
SEMESTER END EXAMINATION, AUGUST 2020.

SEMESTER : V(Repeat) of B.SC.	**DATE : 12/08/2020**	**TOTAL MARKS: 30**
SUBJECT : MATHEMATICS	**PAPER NAME AND PAPER CODE : Algebra (MTC 105)**	**DURATION : 10 a.m. -12noon**

No. OF PAGES: 01

Instructions:

1. All Questions are Compulsory, however internal choice is available.
2. Figures to the right indicate full marks allotted to questions/sub questions.
3. Use of Non-programmable calculator is allowed.
4. Student shall write down the answers and should sign each and every page with date and then upload the scanned copy/photograph of the answer sheet in PDF format. A student must upload their answer scripts by 2.00 pm.
5. PDF should be titled as: Name of the student, Seat Number and paper name.

Q1. Answer any <u>five</u> of the following: 2x5=10

a) Give two reasons why the set of odd integers under addition is not a group.
b) Let H be a nonempty finite subset of a group G. If H is closed under the operation of G, then H is a subgroup of G.
c) Find an example of a noncyclic group, all of whose proper subgroups are cyclic.
d) Find an isomorphism from the group of integers under addition to the group of even integers under addition.
e) Let $\beta = (1\ 2\ 3)(1\ 4\ 5)$. Write β^{99} in disjoint cycle form.
f) Give an example of a ring that has exactly two maximal ideals.
g) Give an example of a subset of a ring that is a subgroup under addition but not a subring.
h) Find the order of all the elements of the group $\mathbb{Z}^*_7$ with respect to multiplication modulo 7.

Q2. Answer any <u>four</u> of the following: 5x4= 20

a) Suppose that H and K are subgroups of G and there are elements a and b in G such that $aH \subseteq bK$. Prove that $H \subseteq K$.
b) Prove that a commutative ring R with unity is a field iff it has no proper ideals.
c) Prove that $H = \{\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} / n \in \mathbb{Z}\}$ is a cyclic subgroup of $GL_2(\mathbb{R})$.
d) Prove that in a group ,$(ab)^2 = a^2b^2$ iff $ab = ba$.
e) For n >1, prove that $|A_n| = \dfrac{n!}{2}$ where A_n denotes alternating group.
f) Let G = { $f:\mathbb{R}\to\mathbb{R}$/ $f(0) = 0$}. Prove or disprove that G forms a group under addition and multiplication of functions.

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