CARMEL COLLEGE OF ARTS, SCIENCE AND COMMERCE FOR WOMEN.

SEMESTER END EXAMINATION, APRIL 2019.

B. Sc. SEMESTER : II
TOTAL MARKS: 80

SUBJECT AND SUB CODE : Matrices and Linear Algebra (DSC 1B)
DATE : 15/04/19
DURATION: 2 Hours

Instructions:

1. All Questions are Compulsory, however internal choice is available.
2. Figures to the right indicate full marks allotted to questions/sub questions.
3. Use of Non-programmable calculator is allowed.

Q1. Answer any four of the following: **4x4=16**

a) Let W_1 and W_2 be two subspaces of a real vector space V. Show that $W_1 \cap W_2$ is a subspace of V.

b) Find a basis of $\mathbb{R}^3$ with $\{(1,1,-1), (0,1,1)\}$ as subset of it.

c) Check if W_1 and W_2 defined as below forms a direct sum of $\mathbb{R}^3$

$W_1 = \{(x,y,z)/\ x=y,\ z=0\}$; $W_2 = \{(x,y,z)/\ x=0\}$

d) Let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a linear transformation such that $T(x, y) = x; \forall\ (x, y) \in \mathbb{R}^2$. Find kernel and nullity of T.

e) Check if $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ defined as $T(x, y) = (x, y, |x|); \forall\ (x, y) \in \mathbb{R}^2$ is linear transformation.

f) Define Range space of a linear transformation. Let $T: V \rightarrow W$ be a linear transformation then show that Range space of T is a subspace of V.

Q2. Answer any four of the following: **4x4=16**

a) Check if the inner product defined as $<\bar{u}, \bar{v}> = x_1^2 - 2x_1y_1 + 3\ x_2y_2$ where $\bar{u} = (x_1, x_2)$ and $\bar{v} = (y_1, y_2)$ forms an inner product space on $\mathbb{R}^2$.

b) For any two vectors $\bar{u}$ and $\bar{v}$ in an inner product space, prove that $|< \bar{u}, \bar{u} >| \leq \|\bar{u}\|\|\bar{v}\|$.

c) Find the eigen values and the corresponding eigen space of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

d) Solve the following system of equations:

$$4x - 2y + 6z = 8$$
$$x + y - 3z = -1$$
$$15x - 3y + 9z = 21$$

e) If $A = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, what are the possible ranks of the matrix A if a & b are real numbers.

f) Let S be the set consisting of vectors $v_1 = (1,2,1)$, $v_2 = (2, 1, -4)$, $v_3 = (3, -2, 1)$ in the inner product space $\mathbb{R}^3$ with usual inner product. (i) Show that S is orthogonal set and is a basis of $\mathbb{R}^3$ (ii) Find the coordinates of vector (7, 1 , 9) related to the basis.

Q3.A) Attempt any one of the following: **(6)**

i. Prove that every finite dimensional vector space has a basis.

ii. Let V be a vector space over a field F. Let W be the subspace of V. Define $V\backslash W = \{W+\bar{x}/\ \bar{x} \in V\}$ where $W+\bar{x} = \{\bar{u} + \bar{x}/\bar{u} \in W\}$. Define: addition $(W+\bar{x}) + (W + \bar{y}) = W+(\overline{x + y})$ and scalar multiplication $a*(W+\bar{x}) = W+(a*\bar{x})$. Prove that $(V\backslash W, +, *)$ is a Quotient space over F.

Q3.B) Answer the following: **(6)**

Let W_1 and W_2 be the two subspaces of a finite dimensional vector space over a field F then prove that W_1+W_2 is also a finite subspace and

$\dim(W_1+W_2) = \dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2)$.

Q4.A) Attempt <u>any one</u> of the following: **(6)**

i. Define linear transformation. show that linear transformation $T: V \to W$ is injective iff $Ker(T) = \{0\}$

ii. Let $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a linear transformation defined as $T(x, y) = (x\cos\theta + y\sin\theta, x\sin\theta - y\cos\theta)$. Show that T is non-singular and find T^{-1}.

Q4.B) Answer the following: **(6)**

State and prove Rank – Nullity Theorem.

Q5. A) Attempt <u>any one</u> of the following: **(6)**

i. If $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$; Find P such that $P^{-1}AP$ is diagonal matrix and hence find A^4.

ii. Reduce the following matrix to the row reduced echelon form and hence find its rank

$$\begin{bmatrix} 1 & 2 & 2 & -1 \\ 3 & 7 & 1 & 2 \\ 0 & 3 & 2 & 5 \end{bmatrix}$$

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Q5.B) Answer the following: **(6)**

Prove that every matrix satisfies its characteristic equation. *Also* verify it for $A = \begin{bmatrix} -1 & 4 \\ 1 & 3 \end{bmatrix}$

Q6. A) Attempt <u>any one</u> of the following: **(6)**

i. In the inner product space of all 2×2 matrices over $\mathbb{R}$ with the inner product defined as $\langle A;B\rangle = trace(B^TA)$. Find the projection of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ along $B = \begin{bmatrix} 1 & 1 \\ 2 & 5 \end{bmatrix}$

ii. Apply Grahm-Schmidt process to obtain an orthonormal set of {(-1,0,1)(1,-1,0)(0,0,1)}

Q6.B) Answer the following: **(6)**

Let $\{v_1, v_2, \ldots, v_n\}$ is an orthonormal set in an inner product space V. let $v \in V$, prove that $\sum_{i=1}^{n} |\langle v, v_i \rangle|^2 \leq \|v\|^2$. Also prove that equality holds if and only if v is the subspace spanned by the vectors in S.

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