ATN – 11

B.Sc. (Semester – V) Examination, October/November 2018
PHYSICS (Paper – II)
Wave Mechanics

Duration : 2 Hours   Total Marks : 80

Instructions : 1) ***All** questions are **compulsory**.*
2) *Figures to the **right** indicate **full** marks.*
3) ***Symbols** have their **usual** meaning unless otherwise stated.*
4) ***Draw** neat diagrams **wherever** necessary.*
5) ***Use** of calculator is **permitted**.*
6) *Given : $h = 6.62 \times 10^{-34}$ J-s; $e = 1.6 \times 10^{-19}$C; $m_e = 9.1 \times 10^{-31}$ Kg.*

1. Answer **any four** of the following : **(4×4=16)**

a) State de-Broglie's hypothesis. Determine the accelerating potential necessary to give an electron a de-Broglie's wavelength of 1A° which is the size of interatomic spacing of atoms in a crystal.

b) For a particle travelling along the x-axis represented by a wave group, show that the product of the uncertainty Δt in the time at which the particle crosses a fixed point on the x-axis and the uncertainty ΔE in the particle energy, is $\Delta E \cdot \Delta t \geq \hbar / 2$.

c) What is a wave group ? How it is produced ? Determine the phase velocity of wave of the de-Broglie's wave corresponding to wavelength $\lambda = h/p$. What is the physical significance of phase velocity ?

d) State the conditions that must be satisfied by a well behaved wave function.

e) State the four postulates of quantum mechanics.

f) Evaluate the first three energy levels of an electron enclosed in a box of width 10A°.

P.T.O.

2. Answer **any four** of the following : **(4×4=16)**

a) With the help of neat diagram, explain qualitatively G.P. Thomson's experiment of electron diffraction.

b) If Ψ_1 and Ψ_2 are the eigen functions of the operator A with the same eigenvalue, then prove that $c_1 \Psi_1 + c_2 \Psi_2$ is also an eigen function of A with the same eigen value, where c_1 and c_2 are constants.

c) Show that for free particle, Heisenberg uncertainty principle can be written as $\Delta\lambda \cdot \Delta X \geq \frac{\lambda^2}{4\pi}$.

d) State the equivalence of kinetic energy and momentum of a particle with corresponding differential operators. Show that this correspondence in equivalent to postulating the Schrödinger's time dependent wave equation.

e) Explain the operation of tunnel diode with respect to the wave mechanical tunneling of the electron wave function across a potential barrier.

f) With the help of an energy level diagram, explain the role of O^+ (raising) and O^- (lowering) operators when they operate on an eigen state of a one dimensional harmonic oscillator.

3. A) a) Explain the particle location by a single slit in the light of Heisenberg's uncertainty principle. **3**

b) The life time of an excited state of a nucleus is about 10^{-12} sec. What is the uncertainty in the energy of gamma ray photon emitted by it ? **3**

OR

x) Show that a wave group description of matter wave leads to an inevitable uncertainty in position as well as the momentum of a system. **3**

y) Prove the following operator equation : **3**

$$\left(\frac{d}{dx} - x\right)\left(\frac{d}{dx} + x\right) = \frac{d^2}{dx^2} - x^2 + 1$$

B) Describe the Davisson and Germer experiment. Show that it gives quantitative confirmation of de-Broglie's hypothesis. **6**

4. A) a) Describe Heisenberg Gamma Ray microscope experiment. How does this thought experiment support the uncertainty principle ? 3

b) Show that the group velocity V_g and the wave velocity V_p are related by the relation : 3

$$V_g = V_p - \lambda \frac{dV_P}{d\lambda}.$$

OR

x) The radius of a typical atomic nucleus is of the order of 5×10^{-15} m. Use the uncertainty principle to place a lower limit on the energy an electron must have if it is to be part of a nucleus. 3

y) For a de-Broglie's matter wave show that, the group velocity equals the particle velocity. 3

B) Derive the classical wave equation of a string stretched along the x-axis and its displacement is along the y axis. 6

5. A) a) A pendulum with a length of 1 metre has a bob with a mass of 0.1 kg. What is the zero point energy ? 3

b) Consider a particle in a finite rectangular potential well :

$$V(x) = \begin{cases} 0, & |x| < a \\ V_0, & |x| > a \end{cases}$$

Set up Schrödinger's time independent equation and show that there are two different classes of solutions having a definite parity. 3

OR

x) What is an eigen value equation ? Find the eigen value of the function $\psi_n(x) = e^{-\frac{x^2}{2}}$ using the operator $\frac{d^2}{dx^2}$. 3

y) Eigen state of a particle in a infinitely deep potential well of width 'L' is given by, $\Psi_n(x) = \sqrt{\frac{2}{L}}\,\mathrm{Sin}\left(\frac{n\pi x}{L}\right)$. Evaluate the expectation value of its momentum <p>. 3

B) Set up the Schrödinger's time independent equation for a linear harmonic oscillator and solve it by operator method. Show that the energy eigen values are $E_n = \left(n + \frac{1}{2}\right)\hbar\nu$. 6

6. A) a) Write down the Schrödinger's time dependent wave equation for a particle in one dimension. Assuming the potential V(x, t) to be time independent, deduce the time independent form of the equation. 3

b) Electron with energy 1.0 eV is incident on a barreir 10.0 eV high and 0.50 nm wide. Find the transmission probability. 3

OR

x) Explain the phenomena of wave mechanical tunneling of a potential barrier and illustrate the same by considering alpha decay. 3

y) Sketch the eigen function $\Psi(x)$ and corresponding probability densities $\Psi(x)^* \Psi(x)$ for the states n = 1, 2 and 3 for a one dimensional harmonic oscillator. 3

B) Set up the Schrödinger's time independent equation for a particle trapped into a three dimensional box of side lengths L_1, L_2 and L_3. The potential inside the box is V a constant and the walls are impenetrable. Discuss the solutions, How does it introduces degeneracy when the side lengths L_1, L_2 and L_3 of the box are equal ? 6
